PHHQP12: List of Speakers with the Title and Abstract of Their Talks

1. **Mekki Aouachria (University of Batna, Algeria)**

   Title: A spin coherent state path integral for Jaynes-Cummings model with a pseudo-hermitian Hamiltonian

   Abstract: We use the coherent state path integral and a angular model for the spin to solve the Jaynes-Cummings model with a pseudo-hermitian Hamiltonian. The propagators are given explicitly as perturbation series. These are summed up exactly. The energy spectrum and the bi-orthonormal basis of states are deduced.

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   M. Aouachria and F. Halimi, to appear in J. Phys. Conf. Ser. (2013).
   M. Aouachria and R. Rekik, AIP Conf. Proc. **1444** , 265 (2012).
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   T. Boudjedaa, A. Bounames, L. Chetouani, and T. F. Hammann, J. Math. Phys. **36**, 1602 (1995)
   C. Cohen-Tannoudji , J. Dupont-Roc and G. Grynberg, Atom-Photon Interactions, Wiley, New York, 1992.
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   A. Mostafazadeh, J. Math. Phys. **43,** 205 and 6343 (2002).
   Y. Ohnuki and T. Kashiwa, Prog. Theor. Phys. **60**, 548 (1978).
   A. M. Perelomov, Generalized Coherent states and Their Application, Springe-Verlag, Berlin, 1986; Commun. Math. Phys. **26**, 222 (1972).

2. **Fabio Bagarello (Palermo University, Italy)**

   Title: Deformed CCR and CAR: Mathematics and physical examples

   Abstract: We consider some deformed versions of CCR and CAR, and we discuss the related functional structure. The arising particles, called *pseudo-bosons* and *pseudo-fermions* respectively, are shown to appear in several models introduced in recent years in the literature on PT-quantum mechanics.

   F. Bagarello, J. Math. Phys. **50**, 023531 (2010); **51**, 053508 (2010); **52**, 063521, (2011);**54**, 023509, (2013); J. Phys. A **43**, 175203 (2010); **44**, 015205 (2011); **45**, 444002, (2012); Phys. Lett. A **374**, 3823 (2010); **376**, 70 (2011); Int. J. Theor. Phys.**50**, 1060 (2011); Rep. Math. Phys., **68**, No. 2, 175 (2011); and "*Pseudo-bosons for the D_2 type quantum Calogero model*," JMAA, to appear.
   F. Bagarello, S. T. Ali, and J. P. Gazeau, J. Math. Phys. **51**, 123502 (2010)

F. Bagarello, A. Inoue, and C. Trapani, J. Math. Phys. **52**, 113508, 2011
F. Bagarello and M. Znojil, J. Phys. A **44** 415305 (2011); **45**, 115311, (2012)

3. **Igor Barashenkov (University of Cape Town, South Africa, and New Zealand Institute for Advanced Study, New Zealand)**

   Title: $\mathcal{PT}$-symmetry breaking in a necklace of coupled optical waveguides

   Abstract: We consider parity-time ($\mathcal{PT}$) symmetric arrays formed by N optical waveguides with gain and N waveguides with loss. When the gain-loss coefficient exceeds a critical value γ_c, the $\mathcal{PT}$-symmetry becomes spontaneously broken. The purpose of this study is to calculate $\gamma_c(N)$. Whether the necklace is closed or open, and no matter whether the waveguides are alternating or clustered, this problem admits an exact solution in terms of zeros of orthogonal polynomials. The expression for the alternating array is particularly simple; namely, $\gamma_c = \sin[\pi/2(2N+1)]$, for the open alternating necklace, and $\gamma_c = \begin{cases} 0, & N = \text{even}, \\ \sin\left(\frac{\pi}{2N}\right), & N = \text{odd}, \end{cases}$ in the alternating periodic case. Finally, we show that in the symmetric phase, the array supports $2N$ solitons with different polarisations.

4. **Hellmut Baumgärtel (University of Potsdam, Germany)**

   Title: The resonance approach of the decay problem in Quantum Mechanics

   Abstract: The talk presents the "crucial semigroup" of the spectral theoretic approach to resonances using Hardy spaces. Its transfer to the physical Hilbert space is pointed out. The transfer explains the difficulty to construct vectors with approximate exponential decay law. The conditions for a time-dependent characterization of the resonances in the Hardy space context are presented, using the "crucial semigroup". Sufficient conditions for the scattering matrix are mentioned, where these conditions are realized.

   H. Baumgärtel, "*The Resonance-Decay Problem in Quantum Mechanics,*" Geometric Methods in Physics. XXX Workshop 2011, Trends in Mathematics, 165-174, Springer, Basel, 2013.

5. **Carl M. Bender (Washington University, St. Louis, US)**

   Title: PT-Symmetric interpretation of double-scaling

   Abstract: The conventional double-scaling limit of an O(N)-symmetric quartic quantum field theory is inconsistent because the critical coupling constant is negative. Thus, at the critical coupling the Lagrangian defines a quantum theory with an upside-down potential whose energy appears to be unbounded below. Worse yet, the integral representation of the partition function of the theory does not exist. It is shown that one can avoid these difficulties if one replaces the original theory by

its PT-symmetric analog. For a zero-dimensional O(N)-symmetric quartic vector model the partition function of the PT-symmetric analog is calculated explicitly in the double-scaling limit.

6. **Arno Bohm (University of Texas at Austin, US)**

   Title: Hamiltonians with complex eigenvalues and asymmetric evolution

   Abstract: The dynamical evolution in standard quantum theory is time symmetric, i.e, for every evolution $U(t)$ of a quantum state by time t, there also exist an inverse evolution $U(-t) = U^{-1}(t)$; and time evolution is represented by a unitary group generated by a self-adjoint Hamiltonian. The same applies also to relativistic quantum systems using unitary representation of the Poincare group. However, this evolution is in conflict with causality, which asserts that a state ρ needs to be prepared in the lab at a finite time t_0, before the Born probabilities $\mathrm{Tr}(A(t)\rho)$ of an observable A can be measured in the state ρ by the counting rates of detectors $N(t)/N$. This means only semigroup transformations $U(t)$ with $t_0 \leq t < \infty$ are physically possible. This time t_0, for which the mathematician would choose $t_0 = 0$, is observed in the lab as an ensemble of finite times t_0^i, $i = 1, 2, 3 \cdots n$. It has been registered in the lab as onset time of dark periods in experiments on single excited ions in a Paul trap, and it represents the time $t_0 = 0$ when the quantum state is prepared. To obtain a theory with a quantum mechanical beginning of time, one cannot use Hermitian (self-adjoint) Hamiltonian in a Hilbert space $\mathcal{H}$. But one has to use a Hamiltonian which is mathematically well-defined as a continuous essentially self-adjoint operator on the space of continuous, rapidly-decreasing Hardy functions $\mathcal{S} \cap \mathcal{H}_{\pm}$.

7. **Vincenzo Branchina (Catania University, Italy)**

   Title: PT-symmetric ϕ^3 quantum field theory

   Abstract: It is shown that a PT-symmetric ϕ^3 quantum field theory in d=6-epsilon dimensions possesses a non trivial fixed point and that the corresponding phase transition is related to the existence of a non-trivial solution of the gap equation. The critical exponents are computed with the help of renormalization-group techniques. It is also shown that, due to its stability property, the PT- symmetric ϕ^3 theory has a higher predictive power than the conventional ϕ^3 theory.

   C. M. Bender, V. Branchina, E. Messina, Phys. Rev. D **85** (2012) 085001
   C. M. Bender, V. Branchina, E. Messina, "*Critical behavior of the PT-symmetric $i\phi^3$ quantum field theory*," Phys. Rev. D., to appear.

8. **Dorje Brody (Brunel University, UK)**

   Title: Geometry of exceptional points and dynamics generated by complex Hamiltonians

Abstract: In this talk I will discuss static and dynamic aspects of complex Hamiltonians. In particular, (a) methods of information geometry are applied to investigate structures of the spaces of eigenstates of complex Hamiltonians, with an emphasis on curvature singularities associated with critical (exceptional) points; and (b) methods of complex analytic geometry are applied to obtain the complex generalisation of Wigner's theorem in quantum mechanics.

9. **Hai Bui (University of Texas at Austin, US)**

   Title: Relativistic time-asymmetric evolution

   Abstract: Relativistic resonances, which are characterized by the width Γ and mass M, can be parameterized in many possible ways such as the gauge dependent-propagator definition in the on-mass shell renormalization scheme or gauge independent-complex pole definition for the scattering energy squared s_R, which can again be written in difference ways. However, in the standard quantum theory, there does not exist a theory in which width of resonance Γ and a lifetime τ of decaying states has exact relations: $\tau = \hbar/\Gamma$. In order to obtain an unified mathematical theory of resonance and decay phenomena, one has to modify the von Neumann's Hilbert space axiom, which consequence is the time-symmetrically dynamical evolution, to new Hardy space axioms. In the relativistic domain, one has to use the interaction-incorporating Poincare transformations to describe the state vectors for decaying relativistic particles. However, due to the Hardy space axiom, one has to restrict to the Poincare semi-group into the forward light cone and obtains not unitary group representation but semi-group representation. Thus, the decaying particles can be described by relativistic Gamow vectors, which is a superposition of the Lippmann-Schwinger kets with Bret-Wigner energy distribution and has the time asymmetrically dynamical evolution $t_0 \leq t < +\infty$.

10. **Emanuela Caliceti (University of Bologna, Italy)**

    Title: An existence criterion for the $\mathcal{PT}$-symmetric phase transition

    Abstract: We consider on $L^2(\mathbb{R})$ the Schrödinger operator family $H(g)$ with domain and action defined as follows

$$D(H(g)) = H^2(\mathbb{R}) \cap L^2_{2M}(\mathbb{R}), \qquad H(g)u = \Big(-\frac{d^2}{dx^2} + \frac{x^{2M}}{2M} - g\,\frac{x^{M-1}}{M-1} \Big) u,$$

    where $g \in \mathbb{C}$, $M = 2, 4, \dots$. $H(g)$ is self-adjoint if $g \in \mathbb{R}$, while $H(ig)$ is $\mathcal{PT}$-symmetric. We prove that $H(ig)$ exhibits the so-called $\mathcal{PT}$-symmetric phase transition. Namely, for each eigenvalue $E_n(ig)$ of $H(ig)$, $g \in \mathbb{R}$, there exist $R_1(n) > R(n) > 0$ such that $E_n(ig) \in \mathbb{R}$ for $|g| < R(n)$ and turns into a pair of complex conjugate eigenvalues for $|g| > R_1(n)$.

11. **Yidong Chong (Nanyang Technological University, Singapore)**

Title: Gain and loss effects in photonic topological insulators

Abstract: This talk discusses the properties of a photonic topological insulator consisting of a lattice of coupled ring resonators, based originally on a proposal by Hafezi et al. [1] It is shown that varying the inter-resonator coupling strengths induces a transition from a trivial to a topological insulator, without the need for aperiodic couplings to generate a synthetic magnetic unit cell [2]; this system thus behaves as a photonic analog of a topological insulator material. The analysis is performed using the transfer matrix method, which has a wider domain of validity for such systems than the Hamiltonian-based tight-binding method. This method also allows the effects of introducing optical gain and loss to be easily studied, including the PT (parity/time-reversal) symmetric case. By incorporating gain and loss into the photonic topological insulator, it can act as a robust optical diode for coupled-resonator waveguide modes.

[1] M. Hafezi, E. A. Demler, M. D. Lukin, and J. M. Taylor, Nature Phys. **7**, 907 (2011).
[2] G. Q. Liang and Y. D. Chong, arXiv:1212.5034.

12. **Francisco Correa (Centro de Estudios Cientificos (CECs), Chile)**

Title: Spectral singularities in PT-symmetric periodic finite-gap systems

Abstract: The origin of spectral singularities in finite-gap singly periodic PT-symmetric quantum systems is investigated. We show that they emerge from a limit of band-edge states in a doubly periodic finite gap system when the imaginary period tends to infinity. In this limit, the energy gaps are contracted and disappear, every pair of band states of the same periodicity at the edges of a gap coalesces and transforms into a singlet state in the continuum. As a result, these spectral singularities turn out to be analogous to those in the non-periodic systems, where they appear as zero-width resonances. Under the change of topology from a non-compact into a compact one, spectral singularities in the class of periodic systems we study are transformed into exceptional points. The specific degeneration related to the presence of finite number of spectral singularities and exceptional points is shown to be coherently reflected by a hidden, bosonized nonlinear supersymmetry.

13. **Giuseppe Della Valle (Politecnico di Milano, Italy)**

Title: A time-periodic $\mathcal{PT}$-symmetric tight-binding lattice Hamiltonian: spectral properties and transport features

Abstract: We investigate the spectral properties and dynamical features of a time-periodic $\mathcal{PT}$-symmetric Hamiltonian on a one-dimensional tight-binding lattice.

Our analysis shows that a high-frequency modulation can drive the system under a transition between the broken-$\mathcal{PT}$ and the unbroken-$\mathcal{PT}$ phases. Interestingly, the time-periodic modulation in the unbroken-$\mathcal{PT}$ regime results in novel spectral features and transport behavior, with no counterpart in Hermitian time-periodic lattices. In particular, a significant broadening of the quasi-energy spectrum is predicted, leading to a *hyper-ballistic* transport regime. Also, near the $\mathcal{PT}$-symmetry breaking the dispersion curve of the lattice band becomes linear, with a strong reduction of quantum wave packet spreading.

C. M. Bender, Rep. Prog. Phys. **70**, 957 (2007).
C. F. M. Faria and A. Fring, J. Phys. A **39**, 9269 (2006).
M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998).
S. Longhi, Phys. Rev. B **80**, 235102 (2009).
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A. Mostafazadeh, J. Phys. A **36**, 7081 (2003).
A. Regensburger, C. Bersch, M.-A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, Nature **488**, 167 (2012).
C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev and D. Kip, Nat. Phys. **6**, 192 (2010).
C. T. West, T. Kottos, and T. Prosen, Phys. Rev. Lett. **104**, 054102 (2010).

14. **Sanjib Dey (City University of London, UK)**

Title: Bohmian quantum trajectories from coherent states

Abstract: We find that real and complex Bohmian quantum trajectories resulting from well-localized Klauder coherent states in the quasi-Poissonian regime possess qualitatively the same type of trajectories as those obtained from a purely classical analysis of the corresponding Hamilton-Jacobi equation. In the complex cases treated the quantum potential results to a constant, such that the agreement is exact. For the real cases we provide conjectures for analytical solutions for the trajectories as well as the corresponding quantum potentials. The overall qualitative behaviour is governed by the Mandel parameter determining the regime in which the wavefuntions evolve as soliton like structures. We demonstrate these features explicitly for the harmonic oscillator and the Poschl-Teller potential.

S. Dey and A. Fring, arXiv:1305.4619

15. **Ahmed Diaf (University of Khemis Miliana, Algeria)**

Title: The Feynman-Kleinert method applied to anharmonic complex-$\mathcal{PT}$ potentials

Abstract: We calculate the energy of the ground state relating to a physical system

described by a complex potential having a $\mathcal{PT}$-symmetry given by :

$$V(x) = A_2x^2 + iA_1x + iA_3x^3 + A_4x^4 + iA_5x^5 + A_6x^6$$

where A_1, A_2, A_3, A_4, A_5 and A_6 are the potential parameters. In order to obtain the energy, we use the Feynman-Kleinert method [1-4] of the Feynman path integral approach [5] to quantum mechanics. This technique is based on the matrix density development on the basis of the harmonic oscillator wave functions, and leads to very acceptable results in the case of anharmonical potentials. We give after, how we can build a complex-$\mathcal{PT}$ potential from a chosen trial wave function. Finally, our results are compared to those given by the QES method, and some special cases are also discussed.

16. **Andreas Fring (City University of London, UK)**

    Title: Non-Hermitian representations for noncommutative spaces with minimal length uncertainty relations

    Abstract: We will report on recent results related to noncommutative spaces with commutation relation amongst their canonical variables which imply minimal lengths relations. We demonstrate the relation to q-deformed oscillator algebras and provide an explicit construction for Klauder coherent states related to non-Hermitian Hamiltonians with discrete bounded below and nondegenerate eigenspectrum. The underlying spacetime structure is taken to be of a noncommutative type with generalized version of Heisenberg's uncertainty relations implying minimal lengths. We represent cases for which the uncertainty relations for the constructed states are shown to be saturated in a Hermitian as well as a non-Hermitian setting. The computed value of the Mandel parameter dictates that the coherent wavepackets are assembled according to sub-Poissonian statistics. Fractional and superrevival times, indicating the superposition of classical-like sub-wave packets are clearly identified.

17. **Manuel Gadella (University of Valladolid, Spain)**

    Title: Models of Resonance Systems

    Abstract: We introduce some models of one dimensional quantum systems showing resonances. These models include point potentials. The location of resonances can be done by available numerical methods.

18. **Gastón García-Calderón (Universidad Nacional Autónoma de México, Mexico)**

    Title: Theory of resonant states for open quantum systems

    Abstract: The formalism of resonant states provides an exact non-perturbative

analytical approach for the description of a broad class of open quantum systems. Resonant states are defined by imposing outgoing boundary conditions to the solutions of the Schrdinger equation. This leads to complex energy eigenvalues and hence to a non-Hermitian formulation. This contribution discusses some rigorous analytical results of this formalism with applications to scattering and decay.

19. **Savannah Garmon (University of Tokyo, Japan)**

Title: Triple level coalescence and Purcell effect in the spectrum of dipole molecules at continuum threshold in a waveguide

Abstract: We demonstrate using examples from AMO physics that a discrete state coupled to continuum in the vicinity of a threshold gives rise to non-analytic effects in the spectrum of the full Hamiltonian. In the case that an inverse square root divergence in the density of continuum states occurs at the threshold (typical of 1D systems) we demonstrate that a particularly rich structure emerges. Specifically, given certain conditions on the interaction potential [1], there will appear three discrete states and four exceptional points (EPs) in the vicinity of the threshold. Two virtual bound states collide to form a resonance, anti-resonance pair at a real-valued EP2 [2], while a bound state exhibits an avoided crossing with the other two states influenced by two complex-valued EP2s. Finally, a real-valued EP3 occurs directly at the continuum threshold at which all three levels coalesce. We also demonstrate that under the influence of the EP3 the decay width of the resonance state exhibits a dramatic amplification in this region, which is an example of the Purcell effect [3]. We use the excitation spectrum of a dipole oscillator traveling in an electromagnetic waveguide as one of our physical models [4].

[1] S. Garmon, H. Nakamura, N. Hatano, and T. Petrosky, Phys. Rev. B **80**, 115318 (2009).
[2] S. Garmon, I. Rotter, N. Hatano, and D. Segal, Int. J. Theor. Phys. **51**, 3536 (2012).
[3] E. M. Purcell, Phys. Rev. **69**, 681 (1946).
[4] T. Petrosky, C -O. Ting, and S. Garmon, Phys. Rev. Lett. **94**, 043601 (2005).

20. **Ananya Ghatak (Banaras Hindu University, India)**

Title: Various aspects of invisibility and reciprocity from a PT-symmetric non-Hermitian potential

Abstract: We complexify a 1-d potential, $V(x) = V_0 \cosh^2 \mu\{\tanh[(x - \mu d)/d] + \tanh(\mu)\}^2$, which exhibits bound, reflecting and free states to study various properties of a non- Hermitian system. This potential turns out to a PT-symmetric one when the parameters μ and/ or d become imaginary and exhibits PT-phase transition. Explicit scattering states are constructed for the complexified version

of this potential. Left and right handednesses are shown for reflecting and transmission coefficients. For some discrete energy of the incident particle reciprocity is restored. For certain values of energy the potential becomes invisible.

21. **Mariagiovanna Gianfreda (Università del Salento, Lecce, Italy)**

Title: Two topics in PT-symmetric quantum mechanics

Abstract: This talk is a summary of two studies on the theory of $PT-$symmetric quantum mechanics. It describes recent results obtained jointly with Carl M. Bender [1,2]. The first result [1] concerns the explicit calculation of infinite unbounded C operators for the quantum harmonic-oscillator Hamiltonian H, satisfying the three algebraic equations $C^2 = 1$, $[C, PT] = 0$ and $[C, H] = 0$. Even though the calculation involves formal summation of infinite series of singular operators, the validity of the formal results is verified by showing that the class of unbounded C operators satisfies the eigenvalue equation $C|\psi_n\rangle = (-1)^n|\psi_n\rangle$, where $|\psi_n\rangle$, are eigenstates of H. The verification is performed by applying a powerful summation technique, *i.e.*, a discrete variant of dimensional continuation that is used to regulate divergent Feynman integrals.

The second result [2] concerns the study of a theoretical model inspired by recent experiments on a $PT-$symmetric system of two coupled optical whispering galleries (optical resonators) [3] . The optical system can be modeled as a pair of coupled linear oscillators, one with gain and the other with loss. If the coupled oscillators have a balanced loss and gain, the system is described by a Hamiltonian that is a generalization of the Bateman's Hamiltonian. This theoretical model exhibits two PT transitions depending on the size of the coupling parameter ϵ. It turns out that the classical and the quantized versions of the system exhibit transitions at exactly the same values of ϵ.

[1] C. M. Bender and M. Gianfreda, J. Phys. A **46** (2013), arXiv:hep-th/1302.7047.
[2] C. M. Bender and M. Gianfreda, arXiv:hep-th/1305.7107.
[3] Experimental work performed by L. Yang, S. K. Ozdemir, and B. Peng at Washington University in St Louis.

22. **Eva-Maria Graefe (Imperial College London, UK)**

Title: Mean-Field approximation for non-Hermitian and PT-symmetric many-boson systems

Abstract: The mean-field description of many-boson systems can be formulated as the effective dynamics resulting from a constraint to fully condensed states. For Hermitian systems of weakly interacting particles this yields the Gross-Pitaevskii equation for the dynamics of the effective single-particle wave function. It has recently been shown that this approximation is modified in the presence of non-

Hermitian terms in the many-particle Hamiltonian, for the example of a Bose-Hubbard dimer with complex on-site energies. Here we review the generalised mean-field approximation for non-Hermitian systems, and compare the resulting dynamics to those arising from a Gross-Pitaevskii equation with complex on-site energies. Further we consider the case of an additional complex particle interaction term, modelling particle losses due to interaction. The latter leads to a complex nonlinear term in the mean-field approximation in contrast to the linear term arising from complex on-site energies. The interplay between non-linearity and non-Hermiticity modifies the self-trapping transition and leads to new bifurcation scenarios that are analysed in detail.

The talk is based on joint work with Hans-Jürgen Korsch (TU Kaiserslautern, Germany), Astrid Niederle (Saarland University, Germany) and Chiara Liverani (Imperial College London, UK)

23. **Uwe Günther (Helmholtz-Center Dresden-Rossendorf, Germany)**

Title: Time-dependent $\mathcal{PT}-$symmetric phase transitions: an exactly solvable model and its underlying geometry

Abstract: A time-dependent 2×2 matrix Hamiltonian is designed which preserves formal $\mathcal{PT}-$symmetry, passes through an exceptional point and whose evolution matrix can be obtained analytically. This exactly solvable model allows for a full analytical treatment of the underlying geometric deformation features of the eigenspace (the structure of the eigenvector bundle) including its geometric and algebraic properties. First results in this direction are presented.

24. **Chao Hang (East China Normal University, Shanghai, China)**

Title: PT-symmetry with a system of three-level atoms

Abstract: We show that a vapor of multilevel atoms driven by far-off resonant laser beams, with possibility of interference of two Raman resonances, is highly efficient for creating parity-time (PT) symmetric profiles of the probe-field refractive index, whose real part is symmetric and imaginary part is anti-symmetric in space. The spatial modulation of the probe-field susceptibility is achieved by proper combination of standing-wave strong control fields and of Stark shifts induced by far-off-resonance laser fields. As particular examples we explore a mixture of isotopes of Rubidium atoms and design a PT-symmetric lattice and a parabolic refractive index with a linear imaginary part.

Phys. Rev. Lett. 110, 083604 (2013).

25. **Naomichi Hatano (University of Tokyo, Japan)**

Title: Complete and bi-orthgonal basis for resonant states

Abstract: In open quantum systems, the bound states together with the integration over the scattering states form a complete orthonormal basis set. In order to take account of resonant states, we would deform the integration contour in the complex wave-number plain. This approach has a significant drawback; there would not be a unique expansion of an arbitrary state with respect to resonant states because one would never get rid of the deformed background integral from the basis set.

We here report a complete and bi-orthonormal basis set which consists of all discrete states (the resonant states, the anti-resonant states, the bound states and the anti-bound states) but does *not* contain the background integral at all. The key ingredient is the quadratic eigenvalue problem. Using the approach of the effective Hamiltonian, we reformulate the Schrödinger equation of an open quantum system in terms of a quadratic eigenvalue problem of a non-Hermitian closed system. This gives a complete and bi-orothnormal basis set of all discrete states under a specific measure.

This is a collaboration with Shachar Klaiman, Gonzalo Ordonez and Kanabu Nawa.

26. **Ulrich D. Jentschura (Missouri University of Science and Technology, Missouri, US)**

Title: Applications of PT Symmetry and Pseudo-Hermiticity:
From the Imaginary Cubic Perturbation to Elementary Particle Physics

Abstract: In recent years, pseudo-Hermitian and/or PT-symmetric Hamiltonians have been established as a valid extension of the concept of Hermiticity. Indeed, pseudo-Hermitian operators are able to describe physical situations where "gain" and "loss" terms in the time evolution compensate each other, and the resulting eigenvalues are real rather than complex. We illustrate this statement on the basis of numerical calculations of eigenstate wave functions for the imaginary cubic perturbation, which is a paradigmetic example of a pseudo-Hermitian operator, and analyze the symmetry properties of the wave functions [1]. Although pseudo-Hermitian operators have proven to facilitate the description of "engineered" physical systems such as PT-symmetric waveguides, one could argue that they have not found any proven application yet on a more fundamental level. We shall present one candidate equation, which is connected with neutrino physics. Indeed, the Dirac equation with a pseudoscalar mass term has recently been proven to yield a pseudo-Hermitian Hamiltonian in the noncovariant formulation [2]. Also, its plane-wave solutions have been shown to fulfill sum rules, hitherto unknown, which facilitate the quantization of the generalized Dirac theory. When applied to neutrino physics, the generalized Dirac equation leads to an alternative explanation of the absence of right-handed neutrino, and left-handed antineutrino states, complementing the seesaw mechanism. Generalized Gordon decompositions, and the physical inter-

pretation of the Noether current in the generalized Dirac theory will be discussed.

[1] U. D. Jentschura and J. H. Noble, e-print arXiv:1301.5758 [quant-ph]
[2] U. D. Jentschura and B. J. Wundt, Eur. Phys. J C **72**, 1894 (2012);
[3] U. D. Jentschura and B. J. Wundt, J. Phys. A **45**, 444017 (2012);
[4] U. D. Jentschura and B. J. Wundt, ISRN High Energy Physics 374612 (2013).

27. **Vladimir Konotop (University of Lisbon, Portugal)**

Title: Nonlinear localized modes in PT-symmetric lattices. Two physical examples

Abstract: A brief survey of the main properties of nonlinear modes in PT-symmetric lattices, with focus on the small amplitude and large amplitude (anticontinuum) limits allowing for analytical consideration, will be given. The theory is applied to two physically relevant examples: nonlinear modes in a PT-symmetric birefringent coupler [1] and the effect of lifting the degeneracy of vortex modes in lattice rings with a PT symmetric defect [2].

[1] K. Li, D. A. Zezyulin, V. V. Konotop, and P. G. Kevrekidis, "*Parity-time symmetric optical coupler with birefringent arms*," submitted.
[2] D. Leykam, V. V. Konotop, and A. S. Desyatnikov, Opt. Lett. **38**, 371 (2013).

28. **Sergii Kuzhel (AGH University of Science and Technology, Krakow, Poland)**

Title: On the Lax-Phillips scattering theory for PT-symmetric Schrödinger operators

Abstract: Nowadays, scattering problems related to PT-symmetric (non-selfadjoint) Schrödinger operators attracts a lot of interests. In the present lecture, I would like to contribute to this inspiring field by studying the scattering of PT-symmetric Schrödinger operator with the use of an operator-theoretical interpretation of the Lax-Phillips approach in scattering theory developed in my works during the last decade.

29. **Félix Maldonado (del Instituto Politécnico Nacional, Mexico)**

Title: Resonance conditions for singular potentials

Abstract: The use of singular potentials in quantum mechanics began with the singular potential proposed by Fermi to modelate the nuclear potential. Later, Faddev made a complete study of the singular potential for the Laplace operator, which became the use of such a singular potentials from the more mathematial point of view, because its usefulness in resolving some problems in mathematical physics. As we know, the standard interpretation of quantum mechanics uses hermitian operators and complete set of vectors to describe a physical system. Some of real physical systems are well approximated, in many cases, by some potentials for

which the Schrödinger equation has an analitical solution. In this case we use a solvable model in one dimension, defined as: a harmonic potential in the negative axis, a singular potential at the origin and no potential for the positive part of the axis. We made use of Green's function to obtain the boundary conditions the wave function must satisfy and reach the same result which is obtained in a paper by Kurasov.

30. **Hossein Mehri-Dehnavi (Babol University of Technology, Iran)**

Title: Pseudo-Hermitian representation of a complex scattering potential with two point interactions

Abstract: A Hamiltonian H is said to be pseudo-Hermitian if its adjoint $H^\dagger$ satisfies $H^\dagger = \eta H \eta^{-1}$, for some Hermitian invertible operator η. Under the assumption of the diagonalizability of H, one can show that its spectrum is real if and only if there exists a positive-definite (metric) operator η [1]. The diagonalizability of an operator is equivalent to the lack of exceptional points and spectral singularities [2]. Here we would like to study the pseudo-Hermiticity of the non-Hermitian scattering Hamiltonian, $\mathrm{H} = p^2/2m + \zeta_- \delta(\mathrm{x}+\alpha) + \zeta_+ \delta(\mathrm{x}-\alpha)$, where $\zeta_\pm$ and α are respectively complex and real parameters and $\delta(\mathrm{x})$ is the Dirac delta function. We try to present a generalization of the perturbative construction of the metric operator for non-Hermitian Hamiltonians with more than one perturbation parameter. Then we will employ this method to construct a (positive-definite) metric operator η and the corresponding equivalent Hermitian Hamiltonian h, for the region at which H is quasi-Hermitian. η turns out to be a (perturbatively) bounded operator for the cases that the coupling constants have opposite sign ($\zeta_+ = -\zeta_- := \zeta$). This in particular contains the $\mathcal{PT}$-symmetric [4] case with purely imaginary couplings ($\zeta = -\zeta^* = \pm\zeta_\pm$). We also calculate the energy expectation values for certain Gaussian wave packets to study the nonlocal nature of h.

This talk is based on Ref. [4]; a joint work with Ali Mostafazadeh (Koç University, Turkey), and Ahmet Batal (Sabancı University, Turkey).

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[3] C. M. Bender and S. Boettecher, Phys. Rev. Lett. **80**, 5243 (1998); C. M. Bender, Rep. Prog. Phys. **70**, 947 (2007).
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31. **Ali Mostafazadeh (Koç University, Turkey)**

Title: $\mathcal{PT}$-symmetric nature of unidirectional invisibility, spectral singularities, and their nonlinear generalization

Abstract: This talk consists of three parts. In its first part, I offer a general introduction of spectral singularities of complex scattering potentials and their physical meaning [1]. In the second part, I discuss the role of $\mathcal{PT}$-symmetry in constructing coherent perfect absorbing lasers [2] and unidirectional invisibility [3]. In part three, I give the definition [4] and outline some of the consequences [5] of a notion of spectral singularity for a class of nonlinear waves with confined nonlinearities.

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[3] A. Mostafazadeh, Phys. Rev. A **87**, 012103 (2013), arXiv:1206.0116.
[4] A. Mostafazadeh, "*Nonlinear Spectral Singularities for Confined Nonlinearities,* arXiv: 1303.2501, Phys. Rev. Lett., to appear.
[5] A. Mostafazadeh, Phys. Rev. A **87**, 063838 (2013), arXiv: 1303.4874.

32. **Jonathan H. Noble (Missouri University of Science and Technology, Missouri, US)**

Title: PT Symmetry, complex inner product, and matrix diagonalization

Abstract: In the numerical calculation of resonance energies $\mathcal{P}T$-symmetric Hamiltonians, one often encounters complex symmetric (non-Hermitian) matrices, $H = H^T$ but $H \neq H^+$ (i.e. $H_{mn} = H_{nm}$ and $\exists H_{nm}$ such that $\mathrm{Im}(H_{nm}) \neq 0$). Such a matrix is obtained by projecting the states of the $\mathcal{P}T$-symmetric Hamiltonian onto a suitable basis of trial states. We find that an obvious generalization of the well known Householder reflections to the complex inner-product, i.e., $\langle\psi|\phi\rangle_* = \int dx\, \psi(x)\, \phi(x)$ (no complex conjugation!), along with a combination of Jacobi and Givens rotations, leads to an efficient numerical algorithm [1]. This observation underlines the importance of the complex inner-product in PT-symmetry [2].

[1] U. D. Jentschura, M. Lubasch, J. H. Noble, in preparation.
[2] U. D. Jentschura and J. H. Noble, e-print arXiv:1301.5758.

33. **Radek Novák (Academy of Sciences of the Czech Republic and Czech Technical University in Prague, Czech Republic)**

Title: Bound states in $\mathcal{PT}$-symmetric layers

Abstract: We consider the Laplacian in a tubular neighbourhood of a hyperplane subject to non-Hermitean PT-symmetric Robin-type boundary conditions. They bring the non-self-adjointness into the problem as the probability current does not vanish on either component of the boundary and the layer therefore behaves as an open system. We analyse the influence of the perturbation in the boundary conditions on the threshold of the essential spectrum using the Birman-Schwinger

principle. Our aim is to derive a sufficient condition for existence, uniqueness and reality of discrete eigenvalues. We show that discrete spectrum exists when the perturbation acts in the mean against the unperturbed boundary conditions and we are able to obtain the first term in its asymptotic expansion in the weak coupling regime.

34. **Michael Ogilvie (Washington University, St. Louise, US)**

Title: PT symmetry and Abelian lattice models

Abstract: A large class of Abelian lattice models with complex actions are of great interest in several areas of physics, including models with non-zero chemical potential. As a consequence of PT symmetry, many of these models can be rewritten in a dual form where the action is real. This gives a solution to the sign problem in a large region of parameter space. The dual models can then be treated using familiar analytic and simulation techniques. For models with Z(N) and U(1) symmetry, the dual models generalize the chiral Potts and Frenkel-Kontorova models. These models are well-known for the existence of spatially-modulated phases and commensurate-incommensurate phase transitions, which can now be associated with PT symmetry breaking.

35. **Şahin K. Özdemir (Washington University, St. Louise, US)**

Title: Parity-time symmetry in coupled optical resonators

Abstract: It is today well-known that non-Hermitian Hamiltonians can have entirely real eigenvalues if they commute with the parity-time (PT) operator, $PTH = HPT$, [1]. Systems respecting the PT-symmetry are interpreted as non-isolated physical systems with carefully balanced loss and gain. When the PT symmetry is broken, such systems undergo phase transitions which are reflected as the emergence of complex eigenvalues. Studies in optics that have been primarily focused on coupled waveguide structures, have not only helped understanding the properties of PT-symmetry but also enabled exploration of the practical applications of PT-symmetric concepts, such as non-reciprocal light transmission and cloaking. In this talk, we introduce yet another optical system which exhibit PT-symmetric properties and provide a versatile platform for testing the concepts and applications [2,3]. This new optical platform is formed by two directly coupled on-chip optical microresonators, one having loss and the other having gain. The lossy resonator is made of silica whereas the gain in the second resonator is provided by rare-earth-ions doped into the silica structure of the resonator. By carefully tuning the coupling strength and the gain-loss ratio, we achieved in demonstrating broken and unbroken PT-symmetry regions in the phase space, and demonstrated examples of how this coupled-resonator system can be used as a building block to control the flow of light in optical systems.

[1] C. M. Bender and S. Bottcher, Phys. Rev. Lett. **80**, 5243 (1998).
[2] C. E. Ruter *et al.*, Nature Phys. **6**, 192 (2010).
[3] A. Regensburger *et al.*, Nature **488**, 167 (2012).

36. **Mauricio Pato (Universidade de São Paulo, Brazil)**

Title: Pseudo-Hermitian random matrices with real eigenvalues

Abstract: In a recent article [1], it has been shown that the so-called β-ensemble constitutes a natural starting point for the construction of an ensemble of random matrices which satisfies the pseudo-Hermiticity condition. In the context of random matrix theories, the β-ensemble of Hermitian tridiagonal matrices generalizes the Wigner Gaussian ensemble in such a way that the parameter β, which in the Gaussian case assumes only the integer values 1,2, and 4, is allowed to have any positive real value. By an appropriate removal of their Hermitian condition, the tridiagonal matrices becomes an ensemble of pseudo-Hermitian random matrices with real eigenvalues. Moreover, asymptotically, that is for large matrices, these eigenvalues occupy in the real axis the same compact support of those of the Hermitian matrices. Regarding their fluctuations, the repulsion between neighboring eigenvalues is modified by the presence of a logarithmic term.

37. **Farrin Payandeh (Payame Noor University, Tabriz, Iran)**

Title: Klein paradox in Krein space

Abstract: In Klein paradox, the anomalous and unexpected numbers of reflected electrons from a potential barrier, has appeared as an obstacle beyond physicists. Since Dirac himself had essentially omitted the negative modes in his solutions, the most recognized explanations for such paradox was the consideration of backwardly moving electrons and transmitting positrons with positive energy. This explanation and the similar ones, have been criticized and under consideration for many years. Some physicists support theories like them, however these are just legitimizations. In this article, we are endeavoring to get rid of the paradox itself. We believe that by maintaining the negative modes (or negative energies), it would be possible to regain the total incidental current and the exact number of electrons, without confronting strange backward travelling electrons. Our progress has based on the Krein quantization, concerning all four of Dirac's solutions. What we are dealing with is that we can validate this possibility that the negative states could be considered as the viable energies. Nevertheless the negative energies in quantum physics has not been related to real physical concepts, however the results in this work, may open doors for further considerations and this is what we are looking for.

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38. **Alois Regensburger (University of Erlangen-Nuernberg, Germany)**

Title: Experiments on optical defects states in a temporal PT lattice

Abstract: We study the evolution of bound states in parity-time (PT) symmetric optical mesh lattices with embedded defects. The experiments are performed in the time domain, using a system of two coupled fiber loops with modulated optical phase, gain and loss. For a defect embedded in a periodic lattice which is in its exact PT phase, we observe a transition from real to complex eigenvalues of the localized mode. Moreover, bound states in the presence of gain are no longer restricted to the band gap of the periodic lattice, but can also lie within the continuum of bands. At PT threshold, an exceptional point gives rise to the formation of a laser-like mode which constantly emits radiation to the surrounding lattice.

A. Regensburger, C. Bersch, M. A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, Nature **488**, 167 (2012).
A. Regensburger, M. A. Miri, C. Bersch, J. Näger, G. Onishchukov, D. N. Christodoulides, and U. Peschel, arXiv:1301.1455.

39. **Vasily N. Rodionov (RGTU, Moscow, Russia)**

Title: Non-Hermitian $\mathcal{PT}$-symmetric quantum mechanics of relativistic particles with the restriction of mass

Abstract: The modified Dirac equations for the massive particles with the replacement of the physical mass m with the help of the relation $m \to m_1 + \gamma_5 m_2$ are investigated. It is shown that for a fermion theory with a γ_5-mass term, the lim-

iting of the mass specter by the value $m_{max} = {m_1}^2/2m_2$ takes place. In this case the different regions of the unbroken $\mathcal{PT}$ symmetry may be expressed by means of the restriction of the physical mass $m \leq m_{max}$, [1]. It should be noted that in the approach which was developed by C. Bender et al. for the $\mathcal{PT}$-symmetric version of the massive Thirring model with γ_5-mass term, the region of the unbroken $\mathcal{PT}$-symmetry was found in the form $m_1 \geq m_2$, [2]. However on the basis of the mass limitation $m \leq m_{max}$ we obtain that the domain $m_1 \geq m_2$ consists of two different parametric sectors: i) $0 \leq m_2 \leq m_1/\sqrt{2}$ -this values of mass parameters m_1, m_2 correspond to the traditional particles for which in the limit $m_{max} \to \infty$ the modified models are converting to the ordinary Dirac theory with the physical mass m; ii)$m_1/\sqrt{2} \leq m_2 \leq m_1$ - this is the case of the unusual particles for which equations of motion does not have a limit, when $m_{max} \to \infty$. The presence of this possibility lets hope for that in Nature indeed there are some "exotic fermion fields". As a matter of fact the formulated criterions [3] may be used as a major test in the process of the division of considered models into ordinary and exotic fermion theories. It is tempting to think that the quanta of the exotic fermion field have a relation to the structure of the "dark matter".

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40. **Stefan Rotter (Vienna University of Technology, Austria)**

Title: Exceptional points in gain-loss structures

Abstract: In the first part of my talk I will speak about an interesting relation between the breaking of PT-symmetry in bounded and in unbounded systems [1]. From this relation we can derive the general result that the symmetry breaking transition in unbounded PT scattering systems is invariant with regard to how strongly the system is coupled to the continuum. In the second part of my talk I will present and discuss first experimental data on pump-induced exceptional points in lasers [2,3].

[1] Ambichl, Makris, Ge, Chong, Rotter, to be submitted.
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41. **Alexander Rush (Imperial College London, UK)**

Title: Quantum dynamics of the Swanson oscillator

Abstract: The non-Hermitian PT-symmetric harmonic oscillator, known as the Swanson oscillator, is a standard example of PT-symmetric quantum mechanics.

It is well known that the system has a purely real spectrum for a wide range of parameter values, the boundaries of which can be easily obtained analytically. Less attention has hitherto been paid to the dynamics of the system. Here we consider the time evolution of an initial Gaussian wave packet. In certain parameter ranges, where the spectrum is purely real, the norm of the quantum state diverges at finite time. This is related to the unboundedness of the metric operator of this system.

42. **Jung-Wan Ryu (Pusan National University, Korea)**

    Title: Multiple exceptional points in a non-Hermitian Hamiltonian

    Abstract: We have investigated the exceptional points which are degeneracies of a non-Hermitian Hamiltonian. In the case that three modes are interacting with each other, even though the parametric evolution of the modes cannot be uniquely determined when encircling more than two exceptional points once, we can recover the initial configuration of the modes by encircling two exceptional points three times or three exceptional points twice. We confirm our expectation by numerically calculating the modes of an open quantum system, two dielectric microdisks, and a 3×3 matrix model. Finally, we discuss the global effects originating from multiple exceptional points.

43. **Luis L. Sanchez-Soto (Max Planck Institut für die Physik des Lichts, Erlangen, Germany, and Universidad Complutense, Spain)**

    Title: Geometrical aspects of PT-invariant transfer matrices

    Abstract: I will show that the transfer matrix for a PT-invariant system, when recast in the appropriate variables, can be interpreted as a point in the (3+1)-dimensional de Sitter space. I use a natural PT-invariant composition law for these matrices and confirm that their action appears as a Lorentz transformation. I will try to elucidate the geometrical meaning of the PT symmetry breaking and suggest that the cosmological event horizon arising in the de Sitter metric can be can be unravelled with a simple optical scheme.

44. **Mustafa Sarısaman (Koç University, Turkey)**

    Title: Spectral singularities and whispering gallery modes in a cylindrical gain medium

    Abstract: We address the optical realizations of the mathematical concept of spectral singularity, and demonstrate its relation to whispering gallery modes in a cylindrical gain medium. We introduce a new class of whispering gallery modes which support a spectral singularity and have a divergent quality factor.

    This is a joint work with Ali Mostafazadeh (Koç University, Turkey)

    A. Mostafazadeh and M. Sarisaman, Phys. Rev. A **87**, 063834 (2003).

45. **Frederik Scholtz (National Institute for Theoretical Physics (NITheP), South Africa)**

Title: Who Chose the Inner Product?-or; Towards an Observable Based Interpretation of Quantum Mechanics

Abstract: We argue that the standard quantum prescription, as first formulated by von Neumann, is ambiguous in that the inner product on a Hilbert space is not unique, but many equivalent inner products and corresponding norms, inducing the same topology, can be defined. This ambiguity impacts directly on the notion of observables, defined as the self-adjoint elements of the C^*-algebra of bounded operators on the Hilbert space. The traditional formulation of quantum mechanics tacitly chooses a specific inner product and admissible set of observables. We argue that this procedure is problematic in a number of physical situations and can even bring us in conflict with the fundamental notion of causality. Instead, we propose that the observables themselves should form the basis for our quantum paradigm and the inner product must follow from them. We formulate the precise conditions under which there is a unique relation between observables and inner product and present several generic physical situations where this relation is not unique. However, we show that this does not impede the predictive power of the formalism and that it indeed reduces to the standard formalism when applied correctly. This formalism is, however, more flexible and can accommodate situations that are problematic in the traditional approach. Several examples of this are discussed and demonstrated through model calculations.

46. **Ali Serpengüzel (Koç University, Turkey)**

Title: Spherical optical microresonators

Abstract: Sphere is the ideal geometry for a resonator. With high quality factor morphology dependent resonances such as geometric resonances or whispering gallery modes, the sphere leads itself to various applications in the optical part of the electromagnetic spectrum. In this talk, we will focus on the properties of spherical resonators and their applications in the visible and near-infrared part of the electromagnetic spectrum.

47. **Andrei Shafarevich (M. V. Lomonosov Moscow State University, Russia)**

Title: Quantization condition on Riemann surfaces and spectral series of non-selfadjoint operators

Abstract: We study spectral series of non-selfadjoint operators (like magnetic induction operator) in semiclassical limit. Asymptotics of the spectrum satisfies quantization conditions on the corresponding Riemann surfaces (complex constant

energy curves). Different cycles of the surface, unlike the selfadjoint case, define different parts of the spectrum (lying on different edges of the corresponding spectral graph).

48. **Petr Siegl (University of Bern, Switzerland)**

Title: The role of pseudospectra in $\mathcal{PT}$-symmetry

Abstract: Spectral properties of $\mathcal{PT}$-symmetric oscillators, e.g. p^2+ix^3 and $p^2+x^2+ix^3$, are well-known and have been studied in many works. The pseudospectrum, related basis properties of eigenfunctions, and intrinsic singularities in the metric operator have been investigated in [1]. We discuss the role of pseudospectra for $\mathcal{PT}$-symmetric models and we particularly focus on the shifted harmonic oscillator, Swanson model, and complex singular perturbations of the harmonic oscillator. We demonstrate that the presented approach can serve as a useful test of a singular behaviour of the metric operator.

The talk is based on the results obtained with D. Krejčiřík (NPI ASCR, Řež, Czech Republic) and M. Tater (NPI ASCR, Řež, Czech Republic) and on the joint work with B. Mityagin (OSU, Columbus, USA).

[1] P. Siegl and D. Krejčiřík, Phys. Rev. D, **86**, 121702(R) (2012).

49. **Andrei Sokolov (Saint-Petersburg State University, Russia)**

Title: Strong minimizability of matrix intertwining operators and symmetry operators for matrix Hamiltonians

Abstract: We study the problem of minimization of a matrix intertwining operator, i.e. the problem of separation from this operator a superfluous factor polynomial in the corresponding matrix Hamiltonian. In this case strong (weak) minimizability correspond to separation of a polynomial with matrix (number) coefficients. We present criterions of strong minimizability of a matrix intertwining operator from the right and from the left. These criterions generalize criterion of weak minimizability of a matrix intertwining operator and criterion of minimizability of a scalar intertwining operator that were found earlier. With the help of the received criterions we find criterion of existence of a constant symmetry matrix for a matrix Hamiltonian and suggest the method of construction of a matrix Hamiltonian and a constant symmetry matrix for this Hamiltonian in terms of a set of arbitrary constant vectors and scalar functions.

50. **Roberto Tateo (University of Torino, Italy)**

Title: Spectral singularities in perturbed conformal field theory

Abstract: In presence of a static pair of sources, the spectrum of low-lying states of a confining gauge theory in D space-time dimensions is described, at large source

separations, by an effective string theory. In the far infrared, the latter model corresponds to a two-dimensional conformal field theory.

The first allowed correction about the infrared fixed point corresponds to a composite field built with the components of the energy-momentum tensor. This irrelevant perturbation is quantum integrable and yields, through the thermodynamic Bethe Ansatz, the energy levels of the string which coincide with the Nambu-Goto spectrum.

This exactly solvable model of quantum field theory does not have a standard ultraviolet fixed point, rather it presents a tachyonic critical point.

51. **Giorgos Tsironis (Univerity of Crete, Greece)**

Title: Nonlinear modes in PT-symmetric metamaterials

Abstract: Synthetic systems with matched gain and loss may form parity-time (PT)-symmetric metamaterials described through non-hermitian Hamiltonians and showing a phase transition in between an exact and a broken phase as a function of the gain/loss power [1]. The PT-symmetry breaking has been experimentally observed in optical lattices [2]. We introduce a PT-symmetric metamaterial consisted of split-ring resonator (SRR) dimers, one with loss and the other with equal amount of gain, coupled magnetically while nonlinearity and gain are introduced through tunnel Esaki diodes. Within the framework of the equivalent circuit model [3], extended for the PT- dimer chain, we investigate the dynamics of charge accumulated in the n-th SRR governed by a coupled set on nonlinear ODE's.

In the absence of nonlinearity, for fixed inter-SRR coupling the propagation bandwidths as a function of the gain/loss parameter show the onset of the PT-phase transition and a resulting band modification. The presence of nonlinearity may induce intrinsic localized modes in the form of discrete breathers with the largest part of the total energy concentrated into two neighboring sites belonging to the same gain/loss dimer [3]. The PT- symmetric nonlinear metamaterial may be used for dynamic tuning in the range of the modified band and switching to the broken phase.

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52. **Haydar Uncu (Adnan Menderes University, Turkey)**

Title: Investigation of quantum jump experiments using Time Asymmetric Quantum Theory

Abstract: Time Asymmetric Quantum Theory which provides a unification of resonance and decay phenomena within a consistent mathematical theory leads to quantum mechanical time-asymmetry characterized by a beginning of time t_0. Quantum jump experiments provide an experimental demonstration of this beginning of time. Therefore, we analyze quantum jump experiments using Time Asymmetric Quantum Theory

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53. **Qing-hai Wang (National University of Singapore, Singapore)**

Title: Random matrix ensembles with reduced symmetry

Abstract: In this talk, we propose an extension of conventional random matrix ensembles by imposing symmetry constraints on Hermitian matrices or parity-time-($\mathcal{PT}$-) symmetric matrices. To illustrate the main idea, we first study 2×2 complex Hermitian matrix ensembles with $O(2)$ invariant constraints, yielding novel level-spacing statistics such as singular distributions, half-Gaussian distribution, distributions interpolating between GOE (Gaussian Orthogonal Ensemble) distribution and half Gaussian distributions, as well as gapped-GOE distribution. Such a symmetry-reduction strategy is then used to explore 2×2 $\mathcal{PT}$-symmetric matrix ensembles with real eigenvalues. In particular, $\mathcal{PT}$-symmetric random matrix ensembles with $U(2)$ invariance can be constructed, with the conventional complex Hermitian random matrix ensemble being a special case. In two examples of $\mathcal{PT}$-symmetric random matrix ensembles, the level-spacing distributions are found to be the standard GUE (Gaussian Unitary Ensemble) statistics or "truncated-GUE" statistics.

J. Gong and Q.-h. Wang, J. Phys. A **45**, 444014 (2012), arXiv:1204.6126.

54. **Güenter Wunner (University of Stuttgart, Germany)**

Title: Hermitian four-well potential as a realization of a $\mathcal{PT}$-symmetric system

Abstract: A $\mathcal{PT}$-symmetric Bose-Einstein condensate can be theoretically described using a complex optical potential. The experimental realization, however, of such an optical potential, which describes the coherent in- and outcoupling of particles, is a nontrivial task. We propose an experiment for a quantum mechanical realization of a $\mathcal{PT}$-symmetric system, where the $\mathcal{PT}$-symmetric currents of a two-well system are implemented by coupling two additional wells to the system, which act as particle reservoirs. In terms of a simple four-mode model we derive conditions under which the two middle wells of the Hermitian four-well system behave *exactly* as the two wells of the $\mathcal{PT}$-symmetric system. We apply these conditions to calculate stationary solutions and oscillatory dynamics. By means of frozen Gaussian wave packets we relate the Gross-Pitaevskii equation to the four-mode model and give parameters required for the external potential, which provides approximate conditions for a realistic experimental setup.

M. Kreibich, J. Main, H. Cartarius, and G. Wunner, arXiv: 1302.2008, Phys. Rev. A, to appear.

55. **Özlem Yeşiltaş (Gazi University, Turkey)**

Title: Non-Hermitian topological insulator Hamiltonians

Abstract: We have studied two dimensional Dirac equation with the spatial modulation of velocity for the topological insulators whose surface states are described by massles Dirac equation. We have given an imaginary velocity barrier and obtained the transmission probability. Moreover, we have examined the bound-state models in the presence of complex vector potentials.

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56. **Konstantin Zloshchastiev (University of KwaZulu-Natal, South Africa)**

Title: Comparison and unification of Lindblad and non-Hermitian approaches to open quantum systems with applications in quantum optics

Abstract: We compare two density-operator approaches to describing the effects of a general dissipative environment, namely, the approach based on the Lindblad or master equation (also referred as the Bloch equation in quantum optics) and the formalism based on introducing the anti-Hermitian terms into the Hamiltonian. We propose the approach which unifies the models of both types and handles them in a consistent unified way. It turns out that this allows us not only to spot the

universal differences between the above-mentioned approaches but also expand the range of dissipative environments which can be accounted for. Using the generic two-mode single-atom laser setup as a test-bed application, the time evolution of such "hybrid" model is formulated in terms of the normalized density operator. One can analytically derive the main observational features of this model which can serve as "footprints" when comparing with experiment.

57. **Miloslav Znojil (Academy of Sciences of Czech Republic, Czech Republic)**

Title: Quantum theory at/near Big-Bang/Big-Crunch

Abstract: The questions of the absence of the observability of the Universe before Big Bang (or after Big Crunch) and of its inflatory initial anomalous expansion (or final collapse) will be addressed via a schematic non-covariant quantum model. It is drastically simplified but its evolution is shown to admit the singularities because the underlying Hilbert space of states is constructed as manifestly time-dependent.

M. Znojil, J. Phys: Conference Series **343** 012136 (2012) .