

# List of Speakers, Title and Abstract of Talks

## 3rd International Workshop on Pseudo-Hermitian Hamiltonians in Quantum Physics

Koç University, Istanbul, June 20-22, 2005  
Sponsored by TÜBİTAK

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## List of Abstracts

**Speaker:** Zafar Ahmed (Bhabha Atomic Research Centre, India)

**Title:** Phase-space trajectories and quantization in PT-symmetric quantum mechanics

**Abstract:** Classical phase space trajectories provide a doorway to quantum and statistical mechanics from classical concepts. We present a way to have them yet again in the complex PT-symmetric quantum mechanics such that phase-space quantization leads to real discrete spectrum. We believe that with this, complex PT-symmetric quantum mechanics passes one of the most stringent tests towards a description that is consistent and compatible with conventional quantum mechanics.

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**Speaker:** Metin Aktaş (Middle East Technical University, Turkey)

**Title:** Exact Supersymmetric Solution of The Klein-Gordon Equation for PT-Symmetric Hulthén Potential

**Abstract:** Exact bound states of PT-/non-PT-symmetric and non-Hermitian Hulthén potential (with the parameter  $q$ ) are investigated. Hamiltonian hierarchy procedure is followed in solving of the Klein-Gordon equation.

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**Speaker:** Ioannis Antoniou (Aristotle University, Greece)

**Title:** Irreversible Non-Hermitian Extensions of Quantum Mechanics

**Abstract:** The mathematical formulation of Quantum Mechanics in terms of Algebras of Hermitian Operators on some separable Hilbert space, leads to two fundamental difficulties, namely: 1) The analytic (perturbative) solution of the eigenvalue problem. This is another manifestation of the classical Poincaré Non-Integrability 2) The explanation of Measurement, Exponential Decay and Irreversibility. Both problems have been addressed by different, more or less ad hoc approximation techniques or by involving physically meaningless. Non-Hermitian Operators on the original Hilbert space. Based on the insight of Prigogine on Irreversibility and Non-Integrability and on the work of Bohm on Gamow vectors and the Rigged Hilbert Space formulation of Quantum Mechanics, we show that both problems admit a natural common solution without approximations or meaningless operators. The solution is obtained by extending (the domain of) the original Hermitian Operators to suitable Rigged Hilbert Spaces. Such extensions are simply there for unstable systems with resonances.

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**Speaker:** Harald Atmanspacher (Institute for Frontier Areas of Psychology, Freiburg, Germany)

**Title:** A relation between physical and psychological time arrows

**Abstract:** There are well-known ways to break the time-reversal symmetry of fundamental physical laws in order to describe irreversible physical processes properly. A decade ago, Huw Price argued, from a philosophical point of view, that a psychological time arrow is often confused with physical irreversibility. Following this argument, we studied how a psychological arrow of time, conceived in terms of a directed sequence of mental representations, is related to stable neuronal assemblies as their physical correlates. We found that causal interactions within neuronal assemblies support their stability while non-causal or anti-causal interactions obstruct their stability. This result suggests that causal interactions among neurons are crucial for the directedness of psychological time. Some possible consequences will be discussed.

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**Speaker:** Bijan Kumar Bagchi (University of Calcutta, India)

**Collaborators:** A. Banerjee and C. Quesne

**Title:** Deformed algebras, position-dependent effective masses and Berry phase

**Abstract:** In the last few years the literature on deformed algebras, in the context of quantum mechanical systems admitting position-dependent masses, is growing rapidly. Here we consider the problem of Berry phase corresponding to a deformed quantum condition and work out explicit expressions for Berry connection and curvature. We also explore applications to shape-invariance both for the simple translation and scaling cases.

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**Speaker:** Hellmut Baumgaertel (University of Potsdam, Germany)

**Title:** Truncated Quantum Evolutions, Hardy Spaces and Resonances

**Abstract:** It is known that for the description of resonances of quantum mechanical scattering systems a so-called truncation of the evolution may be useful. The talk presents an ansatz for a truncation which is connected with the concept of Hardy spaces. By this ansatz one is able to establish the corresponding semigroup in a natural way, using some facts of the well-known Lax-Phillips scattering theory.

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**Speaker:** Carl Bender (Washington University, St. Louis, USA)

**Title:** Ghost Busting: Making Sense of Non-Hermitian Hamiltonians

**Abstract:** The Lee model was introduced in the 1950s as an elementary quantum field theory in which mass, wave function, and charge renormalization could be performed exactly. In early studies of this model it was found that there is a critical value of  $g^2$ , the square of the renormalized coupling constant, above which  $g_0^2$ , the square of the unrenormalized coupling constant, is *negative*. For  $g^2$  larger than this critical value, the Hamiltonian of the Lee model becomes non-Hermitian. In this non-Hermitian regime a new state appears whose norm is negative. This state is called a *ghost*. It has always been thought that in this ghost regime the Lee model is an unacceptable quantum theory because unitarity appears to be violated. However, in this regime while the Hamiltonian is not Hermitian, it does possess  $\mathcal{PT}$  symmetry. It has recently been discovered that a non-Hermitian Hamiltonian having  $\mathcal{PT}$  symmetry may define a quantum theory that is unitary. The proof of unitarity requires the construction of a time-independent operator called  $\mathcal{C}$ . In terms of  $\mathcal{C}$  one can define a new inner product with respect to which the norms of the states in the Hilbert space are positive. Furthermore, it has been shown that time evolution in such a theory is unitary. In this talk the  $\mathcal{C}$  operator for the Lee model in the ghost regime is constructed in the  $V/N\theta$  sector. It is then shown that the ghost state has a positive norm and that the Lee model is an acceptable unitary quantum field theory for all values of  $g^2$ .

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**Speaker:** Hynek Bila (Nuclear Physics Institute, Academy of Sciences, Czech Republic)

**Collaborators:** B. Bagchi, V. Jakubský, S. Mallik, C. Quesne and M. Znojil

**Title:** PT-Symmetric Square Well Inside a Real One

**Abstract:** The determination of the spectra of the first few simplest PT-symmetric square-well models is reviewed and discussed. Main attention is paid to the simplest weakly non-Hermitian model with the range  $l$  of imaginary interaction. The construction of the compact secular equation and an efficiency of its graphical re-interpretation will be outlined in detail. Our bound-state solutions remain valid in the limit of the vanishing imaginary interaction  $l \rightarrow 0$  as well as in the opposite, strongly non-Hermitian long-range extreme.

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**Speaker:** Arno Bohm (University of Texas at Austin, USA)

**Title:** From complex energy to time asymmetry for quantum systems

**Abstract:** Standard quantum theory uses self-adjoint Hamiltonians and the Hilbert space as the space of states and of observables. This leads to real energy eigenvalues and spectra. In contrast, a successful description of scattering and decay phenomena uses S-matrix poles of complex energy, incoming and outgoing Lippmann-Schwinger kets with infinitesimal imaginary energy parts and Gamow vectors with complex eigenvalues of the self-adjoint Hamiltonian. Combining these phenomenological notions with mathematical consequences of the Hilbert space axiom leads to well known conflicts (exponential catastrophe, deviations from exponential decay law, problems with Einstein causality). All this can be overcome and a theory that unifies resonance scattering and decay can be obtained, if one uses a new, Hardy space axiom that distinguishes between prepared in-states and detected out-observables. This leads to semigroup time evolution and beginnings of time for individual micro systems, which have been observed in experiments with single laser cooled ions.

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**Speaker:** Emanuela Caliceti (University of Bologna, Italy)

**Title:** On the spectra of a class of PT-symmetric quantum non linear oscillators

**Abstract:** Some further results are described on the reality of the spectrum of PT-symmetric Schroedinger operators, obtained by perturbing a class of harmonic oscillators by means of suitable relatively bounded perturbations, also in dimension greater than one.

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**Speaker:** Valerio Cappellini (University of Trieste, Italy)

**Title:** Classical Limit of Quantum Dynamical Entropies

**Abstract:** Non-commutative dynamical entropies are studied in connection with the classical limit. For systems with a strongly chaotic classical limit, the Kolmogorov-Sinai invariant is recovered on time scales that are logarithmic in the quantization parameter. These quantum systems are not chaotic for any given finite quantization parameter, but become such after the classical limit has been performed; quantum entropy production analysis on logarithmic time scale provides us with a method to test their chaoticity directly on the quantum domain. The model of the quantized hyperbolic automorphisms of the 2-torus is examined in detail. The used techniques are based on a particular class of Coherent States on the torus, fulfilling a very useful dynamical localization property.

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**Speaker:** Maurice Courbage (University of Paris, France)

**Title:** Friedrichs models, Kaon decay-regeneration and irreversibility

**Abstract:** We present calculation the decay of a new survival probability in a Friedrichs model using time operator in the Liouville space of density matrices. The construction provides a probabilistic interpretation of the resonant states characterized in terms of the Hardy class of functions. We also apply the Friedrichs model to give an interpretation of the Kaon decay-regeneration and CP violation.

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**Speaker:** Ciprian Dariescu (Al. I. Cuza University, Romania)

**Collaborators:** Marina-Aura Dariescu and Gabriel Murariu

**Title:** Gravitoelectromagnetically Induced Transitions in Charged Boson Nebulae

**Abstract:** Using the first-order approximating solutions to the system of Klein-Gordon-Maxwell-Einstein equations, describing a minimally coupled charged boson to a spherically symmetric spacetime, we focus on the feedback of gravity and electric field on the charged scalar source. Within a first-order perturbative approach, we generalize the results obtained in our previous works and write down the effective potential and the current, for the non-vanishing momentum case. These can be actually employed in computing quantum transitions, such as the ones related to gravitoelectric particle creation. The extremely short time-constant is confirming the idea that intense gravitational bursts are accompanying the boson star formation.

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**Speaker:** Marina-Aura Dariescu (Al. I. Cuza University, Romania)

**Collaborators:** Ciprian Dariescu and Gabriel Murariu

**Title:** Topologic Quantum Dynamics of Charged Bosons

**Abstract:** Starting with the Nielsen-Olesen Lagrangian, we derive the system of Euler-Lagrange equations, in cylindrical coordinates, for a constant magnetic field. Next, we focus on the term in the vector-potential which expresses pure topological effects. We derive the solution of the Klein-Gordon equation, depending on the Aharonov-Bohm flux and compute the corresponding current density. As it turns out, in the case of integer flux quanta, the radial and azimuthal parts in the scalar modes decouple. Moreover, a radially non-homogeneous azimuthal current can be actually induced by a potentially-based electric field, of pure quantum origin.

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**Speaker:** Rafael de la Madrid (Universidad del Pais Vasco, Spain)

**Title:** On the analytic continuation of the Lippmann-Schwinger eigenfunctions to the resonance energies

**Abstract:** We show that the Gamow-Siegert bras and kets can be obtained through the analytic continuation of the Lippmann-Schwinger bras and kets. We also show that such analytic continuation associates two Gamow-Siegert bras and two Gamow-Siegert kets with each resonance energy. We compare the two Gamow-Siegert kets associated with a resonance pole and show that, in the wave-number representation, they are given by the complex delta function and by the residue distribution. The Gamow-Siegert bras and kets can be used as basis vectors to produce expansions of wave functions. The truncations of such expansions leads to effective, complex Hamiltonians whose eigenvalues are also complex.

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**Speaker:** Ersan Demiralp (Boğaziçi University, Turkey)

**Title:** Properties of Pseudo-Hermitian Hamiltonian for Harmonic Oscillator Decorated with Dirac Delta Interactions

**Abstract:** We have investigated solutions of the Schrödinger equation for one-dimensional harmonic oscillator potential together with even number of Dirac delta functions. These point interactions are located at points  $x = x_i$  and  $x = -x_i$  ( $i = 1, 2, \dots, N$ ) and they have complex conjugate strengths  $\tilde{\sigma}_i$  and  $\tilde{\sigma}_i^*$  respectively. We present explicit forms of eigenfunctions and algebraic eigenvalue equation and numerical solutions for this  $\mathcal{PT}$ -symmetric Hamiltonian.

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**Speaker:** Alvaro de Souza Dutra (UNESP-Campus de Guaratinguetá-DFQ, Brazil)

**Collaborators:** C. Berteli Cardoso, J. A. de Oliveira, and O. C. Winter

**Title:** Classical behavior for Non-Hermitian quantum systems with real energies

**Abstract:** A usual way for quantizing a system is that of constructing the classical Hamiltonian and, then, obtaining its quantum version through a variety of possible approaches. However, the case of non-Hermitian quantum systems with real spectrum is characterized by a complex classical Hamiltonian, in such a way that it is harder than usual to get a simple physical interpretation of these systems. In this work we try a quite different route. We start from the quantum non-Hermitian Hamiltonian, construct a corresponding stationary coherent state, with the special feature of having its highest probabilities accompanying the classical trajectory, and finally try to present a clue of which kind of classical system could be represented quantum mechanically through a non-Hermitian Hamiltonian with real energies.

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**Speaker:** Harun Eğrifes (Ege University, Turkey)

**Collaborator:** Ramazan Sever

**Title:** The Dirac exact bound-state solutions of generalized Hulthén potential in complex quantum mechanics by the Nikiforov-Uvarov method

**Abstract:** In recent years there has been considerable work on pseudo-Hermitian Hamiltonians. Among this kind of Hamiltonians, much attention has been devoted to the investigation of properties of so-called  $\mathcal{PT}$ -symmetric Hamiltonians. In this study we have investigated the reality of exact bound states of  $\mathcal{PT}$ -symmetric and non- $\mathcal{PT}$ -symmetric generalized Hulthén potential. The one-dimensional Dirac equation has been solved by using the Nikiforov-Uvarov method which is based on solving the second-order linear differential equations by reduction to a generalized equation of hypergeometric type. Thus, negative and positive energy states have been discussed for any given set of complex parameters, and the corresponding wave functions have been established.

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**Speaker:** Shao-Ming Fei (University of Bonn, Germany)

**Title:** On integrability and pseudo-Hermitian systems with spin-coupling point interactions

**Abstract:** We study the pseudo-Hermitian systems with general spin-coupling point interactions and give a systematic description of the corresponding boundary conditions and the spectrum properties, bound states for self-adjoint,  $\mathcal{PT}$ -symmetric systems and systems with real spectra. The corresponding integrability for both bosonic and fermionic many-body systems are investigated in detail for the spin-1/2 case. The quantum entanglement and possible applications related to the above systems are discussed.

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**Speaker:** Lidia Ferreira (CFIF, IST, Portugal)

**Title:** Bound states and resonances in mesoscopic systems

**Abstract:** Bound states and resonances are discussed for systems, that can simulate mesoscopic devices. When the proper boundary conditions are imposed to the solution of the Schrödinger equation, the behaviour of the wave function becomes sensitive to the shape of the device. It is the aim of the present study to discuss such effects in conduction channels.

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**Speaker:** Hendrik Geyer (University of Stellenbosch, South Africa)

**Collaborator:** Izak Snyman

**Title:** The Richardson Hamiltonian in the strong coupling limit: new results from an application of the non-unitary Dyson mapping

**Abstract:** Applications of the generalized Dyson mapping lead to non-hermitian Hamiltonians when expressed in the convenient space of independent bosons and fermions. We show how this formalism may be applied to the Richardson model which has recently been shown to be a good model of superconducting metallic grains. For the first time a perturbative calculation of the wave function and a time-dependent analysis become possible, very much facilitated by a proper application of degenerate perturbation theory for non-hermitian operators. We briefly discuss the implications for the description of ultra-cold Fermi gases.

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**Speaker:** Hüseyin Göksu (Süleyman Demirel University, Turkey)

**Title:** Quantum Theory and Interacting Ambiguities

**Abstract:** A recent article by Low shows that ambiguity defined as a single idea perceived in two self consistent but incompatible frames of reference - can be shown to explain much of the phenomena related to dynamics of consciousness. This definition of ambiguity is almost identical to Koestler's definition of creativity and Low states that it can be taken as the first stage to emerge out of nonreflected awareness within the spectrum of thought. The logic of ambiguity consequently defined subsumes classical logic and looks very similar to quantum logic. In this logic, principle of the excluded middle becomes the principle of incompatible frames of reference. Bohr's Principle of Complementarity, which was elevated to the level of logic by Nadeau and Kafatos by their introduction of three criteria, can be shown to be a special case of Ambiguity Logic with the introduction of time. Taking ambiguity as a building block and studying the interaction of multiple ambiguities show how creative patterns can be possible and how this gives a heuristic viewpoint for the interpretation of quantum mechanics. Symmetric and asymmetric visual ambiguities are studied to reflect on these characteristics of ambiguity patterns. The power of Quantum theory, therefore, can be found to lie in the fact that it preserves the ambiguities until the very end of the calculations by avoiding shrinking the abstract probability function which has all the potential of keeping the information for multiple multidimensional ambiguities. According to this viewpoint, quantum theory lets us observe various multidimensional ambiguities to interact freely, and come to creative solutions themselves through self organization. Because its geometrically representable nature, this view of Quantum Mechanics would be very effective to study relativistic viewpoints. The change of local intentions when ambiguities interact looks similar to curvature of space/time in relativity and both attractive and repulsive forces can be shown to be an attractive force towards the dynamically stable ambiguities which is very easy to represent geometrically. The distribution of symmetry axes of the sub-ambiguities interacting within a pattern, also gives a nice framework to study the stability of ambiguity patterns. These and many other intuitive points - which require further research - give us clues that the Theory of Interacting Ambiguities, can be a good framework to study Quantum Mechanics especially its relativistic problems.

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**Speaker:** Bülent Gönül (Gaziantep University, Turkey)

**Title:** A New Algebraic Approach to Perturbation Theory

**Abstract:** An algebraic non-perturbative approach is proposed for the analytical treatment of Schrödinger equations with a potential that can be expressed in terms of an exactly solvable piece with an additional potential. Avoiding disadvantages of standard approaches, new handy recursion formulae with the same simple form both for ground and excited states have been obtained. As an illustration the procedure, well adapted to the use of computer algebra, is successfully applied to quartic anharmonic oscillators by means of very simple algebraic manipulations. The trend of the exact values of the energies is rather well reproduced for a large range of values of the coupling constant. The formalism is also extended to scattering domain.

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**Speaker:** Uwe Guenther (Rossendorf Research Center, Germany)

**Title:** Krein space related physics: PT-symmetric Quantum Mechanics, MHD  $\alpha^2$ -dynamoes, planar Couette flows, ...

**Note:** The talk is based on and extends studies with Frank Stefani and Miloslav Znojil published in math-ph/0501069 (to appear in J. Math. Phys.).

**Abstract:** Operator theoretic structures are discussed which underlie PT-symmetric Quantum Mechanics (PTSQM), the spherically symmetric  $\alpha^2$ -dynamo of magnetohydrodynamics (MHD) and the plane Couette flow of hydrodynamics (described by the Squire equation). Mathematically, the three types of models are closely related as spectral problems in Krein spaces — Hilbert spaces with an indefinite metric structure. In contrast to the purely real spectrum of self-adjoint operators in "usual" Hilbert spaces, the spectrum of self-adjoint operators in Krein spaces consists, in general, of two types of spectral sectors: sectors with purely real eigenvalues and other sectors with pairwise complex conjugate eigenvalues. Transitions between different sectors occur at exceptional points of square root branching type. Knowing the boundaries of the sectors in parameter space one would know, e.g., the boundaries of the physical sectors of PTSQM (with exact/unbroken PT-symmetry) or of the oscillatory regimes of  $\alpha^2$ -dynamos. The underlying Krein space related structure of the different physical setups indicates a possible fruitful interplay of these models in handling some of their technical and conceptual aspects on a unified footing. We make several aspects of such an interplay explicit and demonstrate its usefulness on the example of a PT-symmetric interpolation model and the Squire equation of hydrodynamics. This allows us to gain a deeper insight into the specifics of the Herbst limit.

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**Speaker:** Dieter Heiss (University of Stellenbosch, South Africa)

**Title:** Global Properties of Exceptional Points in some Many Body Models

**Abstract:** The Lipkin model is a popular toy model, first used in nuclear physics, to understand quantum phase transitions including symmetry breaking. However, the thermodynamic limit, that is the limit of large particle numbers, appears rather elusive. The pattern of the exceptional points of the model, in particular their behaviour with increasing particle numbers, is presented. They may give a clue as to the properties of the Lipkin Hamiltonian in the thermodynamic limit.

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**Speaker:** Enriqueta Hernández (UNAM, Mexico)

**Collaborator:** A. Jáuregui and A. Mondragón

**Title:** Crossings and anticrossings of energies and widths of a doublet of unbound states

**Abstract:** The experimentally established universal character of the crossing-anticrossing properties of the energies and widths of an isolated doublet of resonances, observed when the control parameters of the system are varied, is explained in terms of the topological properties of the universal unfolding of the degeneracy point of the two unbound states in parameter space. This result generalizes a theorem of von Neumann and Wigner from bound states to the case of unbound states. We will illustrate these results with a numerical computation of the surfaces representing the complex resonance energy eigenvalues and the unfolding of the degeneracy point of an isolated doublet of resonances as function of the control parameters of the system in the scattering of a beam of particles by a double barrier potential with two regions of trapping.

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**Speaker:** Mikhail Ioffe (Sankt-Petersburg State University, Russia)

**Title:** SUSY-intertwining relations for 2-dimensional non-hermitian models without separation of variables

**Abstract:** Two new methods - SUSY-separation of variables and 2-dim shape invariance - are used to study the properties of some 2-dim models with complex potentials, which are not amenable to conventional separation of variables. These models are partially solvable: a part of energy eigenvalues (both real valued and complex conjugated pairs) and corresponding wave functions are found explicitly.

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**Speaker:** Vit Jakubsky (UJF, AVCR, Czech Republic)

**Collaborator:** M. Znojil

**Title:** An explicitly solvable model of the spontaneous PT-symmetry breaking

**Abstract:** In a schematic two-parametric representation of a generic PT-symmetric system on a finite interval with Dirichlet boundary conditions we contemplate the pair of the purely imaginary delta-function potentials of strength  $\pm\xi$  located at the points  $\pm a$ . Employing the exact solvability of such a model we show that it exhibits nicely the expected quantitative features of the unavoided level crossings (at certain smaller  $\xi_{cross}(a)$ ) and of a "phase-transition" complexification of the energies (at a certain larger, critical  $\xi_{crit}(a)$ ). Combining analytic and perturbative techniques we investigate the strength-, position- and excitation-dependence of its spectrum and of the related "physical" metric operator in Hilbert space.

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**Speaker:** Hugh Jones (Imperial College, UK)

**Title:** On Pseudo-Hermitian Hamiltonians and their Hermitian Counterparts

**Abstract:** In the context of two particularly interesting non-Hermitian models in quantum mechanics we explore the relationship between the original Hamiltonian  $H$  and its Hermitian counterpart  $h$ , obtained from  $H$  by a similarity transformation.

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**Speaker:** Avinash Khare (Institute of Physics, Bhubaneswar, India)

**Collaborator:** Uday Sukhatme

**Title:** New PT-Invariant Periodic Potentials with a Finite Number of Band Gaps

**Abstract:** Associated Lamé potentials  $V(x) = a(a+1)msn^2(x, m) + b(b+1)msn^2(x + K(m), m)$  are used along with supersymmetry and the anti-isospectral transformation  $x \rightarrow ix + \beta$  (where  $\beta$  is any nonzero real number) to construct several new, complex, PT-invariant, periodic potentials with a finite number of band gaps, when the two parameters  $a, b$  are integers. Further, we obtain the band edges as well as the mid-band states for PT-invariant potentials given by  $V(x) = -a(a+1)msn^2(y, m) - b(b+1)msn^2(y + K(m), m) - f(f+1)msn^2(y + K(m) + iK'(m), m) - g(g+1)msn^2(y + iK'(m), m)$  where  $y = ix + \beta$ , obtained from a generalized variant of the associated Lamé potential involving four parameters  $a, b, f, g$ . We show that many of these potentials with integral values of  $a, b, f, g$  are periodic problems with a finite number of band gaps. As a byproduct, we also obtain several new quasi-periodic solutions of Heun's equation.

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**Speaker:** Oleg Kirillov (Moscow State Lomonosov University, Russia)

**Collaborators:** A. A. Mailybaev and A. P. Seyranian

**Title:** Unfolding of eigenvalue surfaces near a diabolic point due to a non-Hermitian perturbation

**Abstract:** This contribution presents a general theory of unfolding of eigenvalue surfaces of real symmetric and Hermitian matrices due to a non-Hermitian complex perturbation near a diabolic point. General asymptotic formulae describing deformations of a conical surface for different kinds of perturbing matrices are derived. As a physical application, singularities of the surfaces of refractive indices in crystal optics are studied.

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**Speaker:** Frieder Kleefeld (CFIF, IST, Portugal)

**Title:** On (non-Hermitian) Lagrangians in (particle) physics and their dynamical generation

**Abstract:** Quantum Field Theory, even though virtually forbidden by L.D. Landau and colleagues for more than a decade due to the unresolved issue of triviality, led in spite of triviality to the theoretical prediction of the electrons magnetic moment, the accuracy of which has not yet been met even by state of the art most refined present experiments. Symanzik's precarious theory, even though virtually declared dead by the 2004's Nobel-prize committee, is not only more than alive since the latest discoveries made within the context of PT-symmetric quantum (field) theory, yet also serves as a most vital candidate in contributing to the explanation of the theory of strong interaction and its unification with the electroweak sector of the standard model of particle physics. It is the purpose of our presentation to explain why this is so. In order to make the case against triviality and towards a non-Hermitian theory of strong interactions we will try to present - besides experimental "evidence" - the concept of "dynamical generation of Lagrangeans" and its implications in a way, which reflects the most recent research results of the author and which is most convenient also for applications beyond particle physics.

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**Speaker:** Ramazan Koç (Gaziantep University, Turkey)

**Title:** Development of an approximate method for quantum optical models

**Abstract:** An approximate method is suggested to obtain analytical expressions for the eigenvalues and eigenfunctions of some quantum optical models. The method is based on the Lie-type transformation of the Hamiltonians. In a particular case, it is demonstrated that Exe Jahn-Teller Hamiltonian can easily be solved within the approximation. The method presented here is conceptually simple and can be easily extended to the other quantum optical models. Possible generalizations of this approach are outlined.

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**Speaker:** Soliman Komy (Helwan University, Egypt)

**Title:** The rigged Hilbert space for simple Quantum systems

**Abstract:** Not submitted.

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**Speaker:** David Krejcirik (Nuclear Physics Institute, Academy of Sciences, Czech Republic)

**Collaborator:** Pedro Freitas

**Title:** Damped wave equation in unbounded domains

**Abstract:** We perform a spectral analysis of the (non-self-adjoint) infinitesimal generator associated with the damped wave equation in unbounded domains. We present some recent results concerning the influence of the essential spectrum on the time-stability/instability of equation solutions.

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**Speaker:** Milos Lokajicek (Institute of Physics, Academy of Sciences of Czech Republic)

**Title:** Quantum mechanics, hidden variables and EPR experiments

**Abstract:** Already in 1933 W. Pauli called attention to time operator problem, which should be interpreted as mutual discrepancy between two basic assumptions of orthodox quantum mechanics (time-dependent Schroedinger equation and superposition principle in the standard Hilbert space). Additional problem (non-unitarity of exponential phase operator) was then demonstrated by Susskind and Glogover in 1964. The hitherto attempts of solving these problems by generalization of Hermitian Hamiltonian or by a simple extension of standard Hilbert space have not been fully successful. Both these deficiencies may be removed, however, by double extension of Hilbert space: first, according to Lax and Philips, and second, according to Fain (see: Lokajicek: <http://arxiv/quant-ph/0501111>, and Kundrat, Lokajicek: Phys. Rev. A 67, 012104, or "New Research in Quantum Physics", Nova Science Publishers Inc., 2004, p. 17). The extended model describes microscopic phenomena as time asymmetric (irreversible) processes, without leading to the known quantum paradoxes. All characteristics following from Schroedinger equations are conserved. The polarization EPR experiments must be then interpreted on the basis of hidden variables (of greater number than in Bell's approach). Some mistaking statements used for supporting the standard quantum-mechanical model will be discussed, as well. Experimental results (Krasa et al.: Phys. Letters A186, 279, 1994) concerning the light transmission through three polarizers and differing fundamentally from the predictions of the standard quantum model may be then described fully in the framework of the extended model.

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**Speaker:** Alexei Mailybaev (Moscow State Lomonosov University, Russia)

**Collaborators:** O. N. Kirillov and A. P. Seyranian

**Title:** Geometric phase around exceptional points

**Abstract:** A wave function picks up, in addition to the dynamic phase, the geometric (Berry) phase when traversing adiabatically a closed cycle in parameter space. We develop a general multidimensional theory of the geometric phase for (double) cycles around exceptional degeneracies in non-Hermitian Hamiltonians. We show that the geometric phase is exactly  $\pi$  for symmetric complex Hamiltonians of arbitrary dimension and for nonsymmetric non-Hermitian Hamiltonians of dimension 2. For nonsymmetric non-Hermitian Hamiltonians of higher dimension, the geometric phase tends to  $\pi$  for small cycles and changes as the cycle size and shape are varied. We find explicitly the leading asymptotic term of this dependence, and describe it in terms of interaction of different energy levels.

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**Speaker:** Bruce Mainland (The Ohio State University at Newark, USA)

**Title:** Relativistic, bound-state solutions with conjugate pairs of complex coupling constants

**Abstract:** Relativistic solutions are obtained to the integral Bethe-Salpeter equation describing bound states of two massive scalars interacting via the exchange of a third, massive scalar. Covariance of the equation implies that the interaction is retarded, and in part because the energy appears twice, a Hamiltonian does not exist. Thus in contrast to the Schrödinger equation, the Bethe-Salpeter equation is solved by specifying the energy and solving for the coupling constant as an eigenvalue. Although the Bethe-Salpeter equation is derived from a Lagrangian in which the coupling

constant is real, depending on the value of the energy and the masses of the scalars, some values of the coupling constant that satisfy the Bethe-Salpeter equation are complex and always occur in conjugate pairs. The unexpected existence of solutions with real energy and a complex coupling constant raises the possibility that there may also be solutions with a real coupling constant and complex energy. The existence of such solutions remains an open question.

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**Speaker:** Nimrod Moiseyev (Technion, Israel)

**Title:** Time-dependent observables in non-Hermitian Quantum Mechanics

**Abstract:** Non-Hermitian Quantum Mechanics (NH-QM) has proved to be an efficient tool in the study of resonance phenomena. The existing Non-Hermitian formalism breaks down when dealing with wavepackets (WP). An open question is how time dependent expectation values can be calculated when the Hamiltonian is non-hermitian? What are the time dependent "bra" states? Using the F-product formalism, which was recently proposed, [J. Phys. Chem., **107**, 7181 (2003)] we calculate the time dependent expectation values of different observable quantities for a simple well known study test case model Hamiltonian. We carry out a comparison between these results with those obtained from conventional QM (i.e., hermitian QM) calculations. The remarkable agreement between those results emphasizes the fact that in the NH-QM, unlike standard QM, there is no need to split the entire space into two regions; i.e., the interaction region and its surrounding. Our results open a gate for new type of WP propagation calculations within the NH-QM formalism that until now were impossible. In particular our work is relevant to the many different fields in physics and chemistry where complex absorbing potentials are introduced in order to reduce the propagation calculations into a restricted region in space while the artificial reflections from the edge of the numerical grid/box are avoided.

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**Speaker:** Alfonso Mondragón (UNAM, Mexico)

**Title:** Local topology and universal unfolding of the energy surfaces at a crossing of unbound states

**Abstract:** We consider the unfolding of a degeneracy point of an isolated doublet of resonance (unbound) energy eigenstates of a time reversal invariant Hermitian Hamiltonian depending on external control parameters. In parameter space, close to the degeneracy point, the real and imaginary parts of the energy eigenvalues are represented by a pair of two-sheeted surfaces with an algebraic branch point of rank one and branch cuts in the real and imaginary parts starting at the exceptional point but extending in opposite directions in parameter space. Associated with this singularity in parameter space, the scattering matrix,  $S_\ell(E)$ , and the Green's function,  $G_\ell^{(+)}(k; r, r')$ , have a double pole in the unphysical sheet of the complex energy plane. Then, the universal unfolding or deformation of a typical degeneracy point of two resonance energy eigenvalues in parameter space is characterized by means of a universal 2-parameter family of functions which is contact equivalent to the pole position function of the isolated doublet of resonances at the exceptional point and includes all small perturbations of the degeneracy conditions up to contact equivalence. Some important observable consequences are: The geometric phase factor acquired by the unbound energy eigenstates when transported adiabatically in a closed double circuit around the exceptional point. The rich phenomenology of crossings and anticrossings of energies and widths, as well as the sudden change in shape of the  $S(E)$ -matrix pole trajectories, observed in an isolated doublet of resonances when one control parameter is varied, which are fully explained in terms of the topological properties of sections of the energy hypersurfaces close to the degeneracy point.

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**Speaker:** Ali Mostafazadeh (Koç University, Turkey)

**Title:** Pseudo-Hermiticity and the Physical Content of  $PT$ -symmetric Quantum Mechanics

**Abstract:** We give a complete description of  $PT$ -symmetric quantum systems defined on the real line or an appropriate complex contour within the framework of Pseudo-Hermitian Quantum Mechanics. In particular, we discuss the construction of the physical observables and the classical limit of these systems.

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**Speaker:** Gheorghe Munteanu (Transilvania University of Brasov, Romania)

**Title:** Gauge Field Theory in Terms of Complex Hamilton Geometry

**Abstract:** On the total space of a G-complex vector bundle  $E$  is defined the gauge transformations. A gauge complex invariant lagrangian determines a special complex nonlinear connection for which the associated Chern-Lagrange and Bott complex connections are gauge invariant. The complex field equations are determined with respect to these associated gauge complex connections. By complex Legendre transformation (the L-dual process) we investigate the

similar problems on the dual vector bundle  $E^*$ . The L-dual Chern-Hamilton and Bott complex connections are also gauge invariant. The complex Hamilton equations are write for the general L-dual hamiltonian obtained as a sum of particle hamiltonian, Yang-Mills and Hilbert-Einstein Hamiltonians.

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**Speaker:** Sami Muslih (Azhar University, Palestine)

**Collaborator:** D. Baleanu

**Title:** About fractional supersymmetric quantum mechanics

**Abstract:** Euler-Lagrange equations are investigated in the presence of the fractional Grassmann variables. The super fractional Hessian was defined and the fractional Hamiltonian of the fractional supersymmetric classical model is constructed.

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**Speaker:** Asiri Nanayakkara (Institute of Fundamental Studies, Sri Lanka)

**Title:** Avoided crossing of eigenstates for non-hermitian and non-hermitian system

**Abstract:** Applicability of classical and quantum perturbation theories near avoided crossing of eigen states of pseudo-Hermitian system are investigated. Comparison between behavior of the eigen states near avoided crossing of 2-D and 3-D Hermitian and non-Hermitian systems are presented.

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**Speaker:** Gor Nikoghosyan (Institute for Physical Research, National Academy of Sciences, Armenia)

**Collaborator:** G. Grigoryan

**Title:** Influence of strong relaxation on electromagnetically induced transparency

**Abstract:** By solving the self-consistent system of Maxwell and density matrix equations to the first order with respect to nonadiabaticity and relaxation, we obtain an analytical solution for the probe pulse propagation. The conditions for efficient storage and propagation of light are analyzed.

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**Speaker:** Christiane Quesne (Universite Libre de Bruxelles, Belgium)

**Collaborators:** B. Bagchi, H. Břla, V. Jakubský, S. Mallik, and M. Znojil

**Title:** Supersymmetry in  $\mathcal{PT}$ -symmetric square wells

**Abstract:** We generalize to the  $\mathcal{PT}$ -symmetric context the well-known supersymmetric hierarchy of certain solvable potentials  $V_j$  initiated by the constant  $V_0 = 0$ . A key to such an extension is found in an appropriate treatment of the discontinuities which characterize all the eligible purely imaginary and piecewise constant non-Hermitian initial  $V_0 \neq 0$ . In detail we show how our new supersymmetric scheme works for the strongly non-Hermitian  $\mathcal{PT}$ -symmetric square well as well as for its weakly non-Hermitian modification. In both cases, the superpotentials  $W_0$ , the partner potentials  $V_1$  and their eigenfunctions are presented in closed form.

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**Speaker:** Valerio Raganelli (European Space Agency, France)

**Title:** The largest class of partial differential equations that may be analytically integrated by using Neumann series

**Abstract:** In this work the broadest possible class of partial differential equations that can be solved in an exact analytical way if and when their solution is represented by a Neumann series, at least in certain space domains, infinitely extended ones, by the way, is found and it is shown that this definition leads to a uniquely determined set of linear partial differential equations. It is thence straightforward to prove that Schrödinger-like and Klein-Gordon-like partial linear differential equations with generally not hermitian, and a fortiori pseudo-Hermitian too, Hamiltonians operators belong to this class.

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**Speaker:** Ingrid Rotter (Max-Planck-Institute, Dresden, Germany)

**Title:** Avoided level crossings and singular points

**Abstract:** The spectrum of an open double quantum dot is studied as a function of different system parameters in order to receive information on the geometric phases of branch points in the complex plane (BPCP). The phases are related to the geometrical phases of the diabolic points (DPs) of the corresponding closed system. The double dot consists of two single dots and a wire connecting them. The two dots and the wire are represented by only a single state each. The spectroscopic values follow from the eigenvalues and eigenfunctions of the Hamiltonian describing the double dot system. They are real when the system is closed, and complex when the system is opened by attaching leads to it. The discrete states as well as the narrow resonance states avoid crossing. The DPs are points within the avoided level crossing scenario of discrete states. At the BPCP, width bifurcation occurs. Here, different Riemann sheets evolve and the levels do not cross anymore. The BPCP are physically meaningful. The DPs are unfolded into two BPCP with different chirality when the system is opened. The geometric phase that arises by encircling the DP in the real plane, is different from the phase that appears by encircling the BPCP. This is found to be true even for a weakly opened system and the two BPCP into which the DP is unfolded.

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**Speaker:** Nasser Saad (University of Prince Edward Island, Canada)

**Collaborators:** H. Ciftci, R. L. Hall

**Title:** Iteration methods for PT-symmetric potentials

**Abstract:** In a previous paper (J. Phys. A **36**, 11807 (2003)), we introduced and developed the ‘asymptotic iteration method’ for solving second- order homogeneous linear differential equations. In this paper, we apply the method to solve Schrödinger’s equation for a variety of complex potentials  $V(x) = (x + C)^2 - \frac{A}{x}$ , of both PT-symmetric  $\Re(A) = \Re(C) = 0$ , and non- Hermitean  $A, C \in \mathbb{C}$  type, which admit real positive eigenvalues. The transition to the Hermitean case  $\Im(A) = \Im(C) = 0$  is discussed, and exact eigenvalues, along with closed-form expressions for the corresponding wavefunctions, for the potentials  $V(x) = (x + \mu)^2 \pm \frac{\lambda}{x}$ ,  $\mu, \lambda \in \mathbb{R}$ , are obtained.

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**Speaker:** Boris Samsonov (Tomsk State University, Russia)

**Collaborators:** B. F. Samsonov, V. Shamsutdinova and A. Pupasov

**Title:** Intertwining (Darboux transformation) operators and complex exactly solvable potentials

**Abstract:** We show that the technique of intertwining (Darboux transformation) operators is a very efficient tool for studying non-Hermitian Hamiltonians. In this respect first we give a brief overview of the method. The main ideas are illustrated by two examples. In the first example we consider transformations between diagonalizable (i.e. Hermitian) Hamiltonians and non-diagonalizable ones keeping the spectrum almost unchanged. In particular, we show how Green functions are transformed during such transformations. In the second example we consider transformations for a one-dimensional Dirac Hamiltonian with a non-Hermitian Hamiltonian of a special form. We show that the considered Dirac equation is equivalent to the non-stationary Schrödinger equation for a two-level system in an external electric field in the rotating wave approximation described by a Hermitian Hamiltonian. This approach permits us to establish an internal pseudosupersymmetry of a two-level system and get new interesting from physical point of view electric fields with a time-dependent frequency for which the Schrödinger equation may be solved analytically. In particular, we discuss a new physical phenomenon consisting in disappearance of the Rabi oscillations.

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**Speaker:** Giuseppe Sclarici (University of Lecce, Italy)

**Title:** Pseudo-Hermiticity and Exceptional Points

**Abstract:** We consider a fermionic time-reversal violating pseudo-Hermitian Hamiltonian which break diagonalizability for some critical parameter values. Starting from this physical example, we highlight some features of the pseudo-Hermitian operators admitting exceptional points.

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**Speaker:** Alexander Seyranian (Moscow State Lomonosov University, Russia)

**Collaborators:** O. N. Kirillov and A. A. Mailybaev

**Title:** Coupling of eigenvalues of complex matrices at diabolic and exceptional points

**Abstract:** The paper presents a general theory of coupling of eigenvalues of complex matrices of arbitrary dimension depending on real parameters. The cases of weak and strong coupling are distinguished and their geometric interpretation in two and three-dimensional spaces is given. General asymptotic formulae for eigenvalue surfaces near diabolic

and exceptional points are presented demonstrating crossing and avoided crossing scenarios. Two physical examples from crystal optics illustrate effectiveness and accuracy of the presented theory.

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**Speaker:** Alireza Shabani (University of Toronto, Canada)

**Title:** Quantum Measurement Approach to a Non-Markovian Master Equation

**Abstract:** It is well known that the Markovian Lindblad master equation can be unravelled in terms of a series of continuous measurements. A relaxation of this measurement procedure is then equivalent to reducing the Markovian constraint. We introduce a probabilistic single-shot measurement process describing a non-Markovian evolution of a system coupled to a bath. The resulting analytically solvable master equation has a form resembling the Nakajima-Zwanzig equation, wherein a memory kernel superoperator characterizes non-Markovian effects. Moreover, we give necessary and sufficient conditions on the kernel function that appears in our equation in order to preserve complete positivity of the dynamical map. We also present physical examples from Cavity QED and Optical lattices as realization of such dynamics.

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**Speaker:** Andrey Sokolov (Sankt-Petersburg State University, Russia)

**Collaborators:** A. A. Andrianov and F. Cannata

**Title:** Non-linear SUSY for non-Hermitian, non-diagonalizable Hamiltonians with continuous spectrum

**Abstract:** The general structure of Polynomial SUSY for complex potentials admitting associated normalizable functions will be elucidated. The restrictions imposed by the PT symmetry will be explained. The generation of non-diagonalizable Hamiltonians with complex potentials from Hermitian ones by SUSY transformation will be described with the special attention to peculiarities of the continuum spectrum. In particular, the interplay between continuum and bound state spectra will be examined in construction of the decomposition of unity. Finally, the role of zero-pseudonorm states in the non-standard time evolution for the decay process will be clarified. All topics will be supported by rigorous theorems and/or key examples.

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**Speaker:** Yossi Strauss (Hebrew University of Jerusalem, Israel)

**Title:** How to define approximate wave functions for scattering resonances in quantum mechanics

**Abstract:** A way of utilizing Lax-Phillips type semigroups for the description of the time evolution of resonances in quantum mechanical scattering problems was recently developed. In the proposed framework resonance poles of the scattering matrix correspond to a decomposition of the time evolution into a background term and an exponentially decaying resonance term evolving according to a semigroup law given by a Lax-Phillips type semigroup; this is called the semigroup decomposition. One of the first models to be analyzed within the new framework is the simple problem of a half-line scattering from a square barrier potential. This model demonstrates clearly the unexpected finding that the new framework allows one to associate with each resonance a definite subspace of the Hilbert space for the scattering problem. This subspace is such that the semigroup decomposition of the time evolution for a vector chosen orthogonal to it does not contain the Lax-Phillips semigroup term corresponding to the resonance. Using this observation we are able to numerically calculate approximate wave functions for the resonances of the square barrier potential.

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**Speaker:** Mohsen Timoumi (Monastir, Tunisia)

**Title:** Subharmonics of a class of Hamiltonian systems

**Abstract:** Consider a relativistic particle with charge  $e$  and rest mass  $m_0$ , moving in a field of a monochromatic wave. The energy is given by :

$$H(t, r, p) = c[m_0^2 c^2 + |p - \frac{e}{c} Re \left( A_0 e^{i(kr - \omega t)} \right)|^2 + Re \left( \varphi_0 e^{i(kr - \omega t)} \right)].$$

and the motion particle is described by the Hamiltonian system :

$$(\mathcal{H}) \quad \dot{x} = JH'(t, x).$$

There have been different works devoted to multiple periodic solutions of  $(\mathcal{H})$  when the hamiltonian  $H$  is periodic in a part of the variables and resonant or coercive at infinity with respect to the other part of variables. Here and under the same assumptions, we are interested in the existence of subharmonic solutions of the Hamiltonian system  $(\mathcal{H})$ , i.e of distinct  $kT$ -periodic solutions of  $(\mathcal{H})$ .

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**Speaker:** Stefan Weigert (University of Hull, UK)

**Title:** Algorithms testing diagonalizability and the reality of the spectrum of finite-dimensional PT-invariant systems

**Abstract:** Contrary to a Hermitean one, a general non-Hermitean operator will neither have a complete set of eigenstates nor real eigenvalues only. Two algorithms are presented which allow one to decide for a PT-invariant operator, acting in a finite-dimensional space, whether (i) its eigenstates form a complete set, and (ii) whether all its eigenvalues are real. In other words, the algorithms check whether a given PT-symmetric matrix is diagonalizable and whether PT-symmetry is ‘broken’ or not.

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**Speaker:** Sujeew Wickramasekara (Rice University, USA)

**Title:** The Generators of Certain Representations of the Causal Poincaré Semigroup

**Abstract:** A theory of relativistic quasistable particles has been recently developed by using a particular class of representations of the causal Poincaré semigroup,  $\P_+$ . This is the semigroup consisting of the proper orthochronous Lorentz transformations and spacetime translations into the forward light cone. The relevant representations of this semigroup are obtained in a suitably defined rigged Hilbert space (RHS). The new theory provides a unifying description of resonances and decaying states, establishes the lifetime-width relation  $\tau = \frac{1}{\Gamma}$  as an exact and universal identity, and brings forth a fundamental principle for defining the mass and width of relativistic resonances. In this talk we will examine some of the algebraic properties of the generators of the above RHS representations of the causal Poincaré semigroup. Unlike those of the unitary representations of the Poincaré group (or, in general, Lie groups) in Hilbert spaces, the generators of the operator semigroup  $\P_+$  do not yield an operator Lie algebra. Instead, they inhabit a cone embedded in the Poincaré Lie algebra, leading to the inference that a clear distinction must be made between the set of operators fulfilling the commutation relations of a Lie algebra and the set of operators integrating to asymmetry transformations. In this light, we point out that the physical meaning of the notion of observable requires careful further analysis in the rigged Hilbert space context.

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**Speaker:** Farhad Zamani (IASBS, Zanjan, Iran)

**Collaborator:** A. Mostafazadeh

**Title:** Pseudo-Hermitian QM and probabilistic interpretation of first-quantized Klein-Gordon Fields

**Abstract:** In this talk we give a consistent formulation of first-quantized relativist quantum mechanics of both real and complex Klein-Gordon fields that would enable one to address the relativistic analogs of typical quantum mechanical problems. We determine the Hilbert space by constructing a one parameter family of conserved current densities  $J_a^\mu$ , with  $a \in (-1, 1)$ , and use the later to yield an explicit manifestly covariant expression for the most general positive-definite and Lorentz-invariant inner product on the space of all solutions of the Klein-Gordon equation. We also obtain an expression for the probability current density  $\mathcal{J}_a^\mu$  for the localization of a Klein-Gordon field in space. We show that the conserved current densities  $J_a^\mu$  are analogous to the chiral current density for spin half fields and in the nonrelativistic limit both  $J_a^\mu$  and  $\mathcal{J}_a^\mu$  tend to the probability current density for the localization of a nonrelativistic free particle in space, but that unlike  $J_a^\mu$  the current density  $\mathcal{J}_a^\mu$  is neither covariant nor conserved. The conservation of  $J_a^\mu$  is related to a global gauge symmetry of the Klein-Gordon fields which is responsible for the conservation of the total probability of the localization of field in space.

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**Speaker:** Miloslav Znojil (Nuclear Physics Institute, Academy of Sciences, Czech Republic)

**Title:** Klein Gordon bag model with the simulated creation/annihilation of quarks

**Abstract:** In the context of the schematic simulations of the quark confinement in baryons, two types of PT-symmetric (or, if you wish, P-pseudo-Hermitian) modifications of the standard and phenomenologically successful non-relativistic bag models are proposed. Firstly, a non-perturbative relativistic kinematics is introduced via transition from Schroedinger to Klein-Gordon equation. Secondly, a “minimal” dynamical simulation of the effects originating from field theory in general and from the implicit presence of the annihilation/creation processes in particular is mediated by the various choices of the locally imaginary mass-term.